Harmonický oscilátor – komplexní reprezentace

harmonický kmit: $x = A\sin(\omega t + \varphi) \longrightarrow Ae^{i(\omega t+\varphi)}$

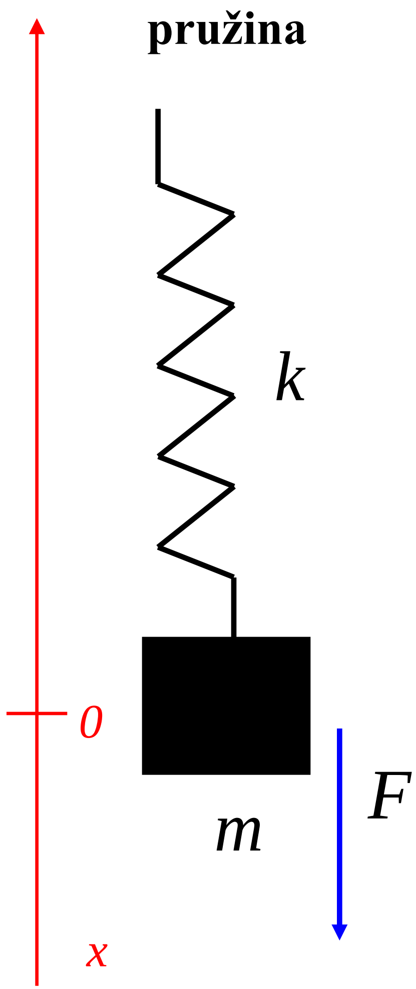
A – amplituda, ω – úhlová frekvence, φ – fázový posuv

$$Ae^{i(\omega t+\varphi)} = A\cos(\omega t + \varphi) + iA\sin(\omega t + \varphi)$$

$$Ae^{i(\omega t+\varphi)} = Ae^{i\varphi}e^{i\omega t} = \hat{A}e^{i\omega t}$$

$\hat{A}$ – komplexní amplituda

Nucené kmity



$$\ddot{x} + \frac{k}{m}x = \frac{F}{m}$$

$$\omega_0^2 \equiv \frac{k}{m}$$

pohybová rovnice

$$\ddot{x} + \omega_0^2 x = \frac{F}{m}$$

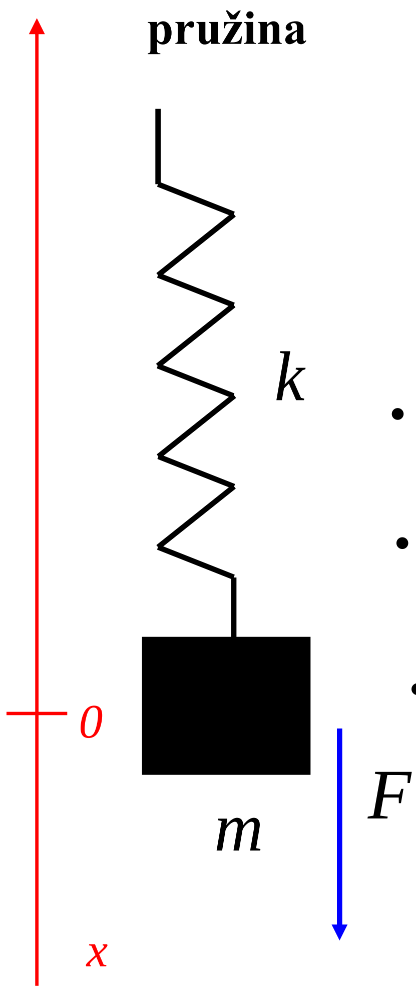
- budící síla: $F = F_0 \sin(\Omega t) = F_0 e^{i\Omega t}$
- obecné řešení: $x = A\sin(\omega t + \varphi) + x_p$ ← partikulární řešení:
- partikulární řešení: $\boldsymbol{x_p} = \hat{x}_p e^{i\Omega t}$

  ↑ komplexní amplituda ($\hat{x}_p$)

$$\hat{x}_p = \frac{F_0}{m(\omega_0^2 - \Omega^2)}$$

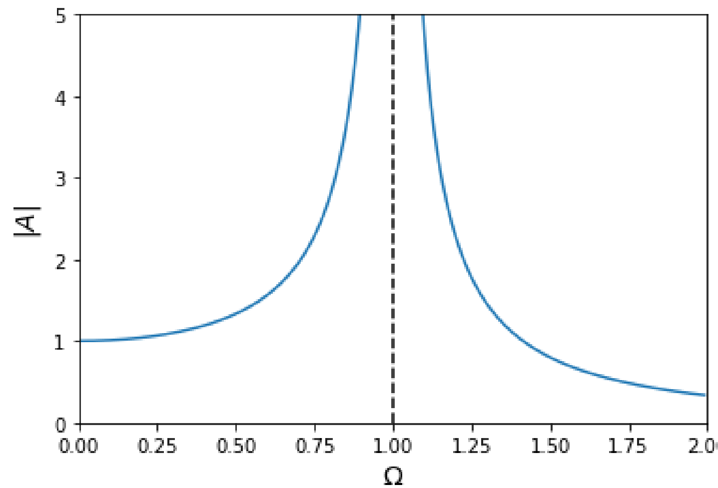
$$x_p = \frac{F_0}{m(\omega_0^2 - \Omega^2)}\sin(\Omega t)$$

Nucené kmity



$$\ddot{x} + \frac{k}{m}x = \frac{F}{m}$$

$$\omega_0^2 \equiv \frac{k}{m}$$

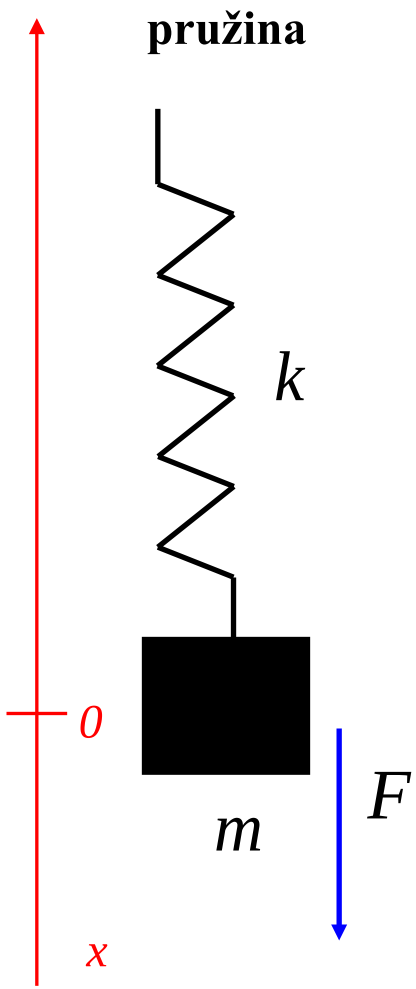


- budící síla: $F = F_0 \sin(\Omega t) = F_0 e^{i\Omega t}$
- obecné řešení: $x = A\sin(\omega t + \varphi) + x_p$ ← partikulární řešení:
- partikulární řešení: $\boldsymbol{x_p} = \hat{x}_p e^{i\Omega t}$ ($\hat{x}_p$ – komplexní amplituda)

$$\hat{x}_p = \frac{F_0}{m(\omega_0^2 - \Omega^2)}$$

$$x_p = \frac{F_0}{m(\omega_0^2 - \Omega^2)}\sin(\Omega t)$$

Nucené kmity s tlumením



$$\ddot{x} + \frac{k}{m}x + \frac{h}{m}\dot{x} = \frac{F}{m}$$

$$\omega_0^2 \equiv \frac{k}{m} \quad 2\delta \equiv \frac{h}{m}$$

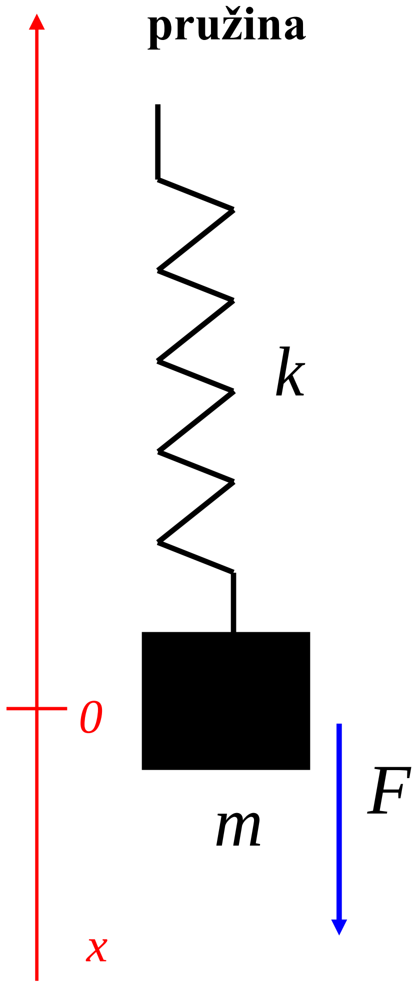
pohybová rovnice

$$\ddot{x} + \omega_0^2 + 2\delta\dot{x} = \frac{F}{m}$$

- budící síla: $F = F_0 \sin\Omega t = F_0 e^{i\Omega t}$
- partikulární řešení: $x_p = A_0 \sin(\Omega t + \vartheta) = A_0 e^{i\vartheta} e^{i\Omega t} = \hat{A} e^{i\Omega t}$
- po dosazení: $\hat{A}\underbrace{(\omega_0^2 - \Omega^2 + 2i\delta\Omega)}_{\boldsymbol{K} = K_0 e^{i\beta}} = \frac{F_0}{m}$

$$K_0 = \left[(\omega_0^2 - \Omega^2)^2 + 4\delta^2\Omega^2\right]^{1/2} \qquad \mathrm{tg}\beta = \frac{2\delta\Omega}{\omega_0^2 - \Omega^2}$$

Nucené kmity s tlumením



$$\ddot{x} + \frac{k}{m}x + \frac{h}{m}\dot{x} = \frac{F}{m}$$

$$\omega_0^2 \equiv \frac{k}{m} \quad 2\delta \equiv \frac{h}{m}$$

$$A_0 K_0 e^{i\vartheta} = \frac{F_0}{m} e^{-i\beta}$$

pohybová rovnice

$$\ddot{x} + \omega_0^2 + 2\delta\dot{x} = \frac{F}{m}$$

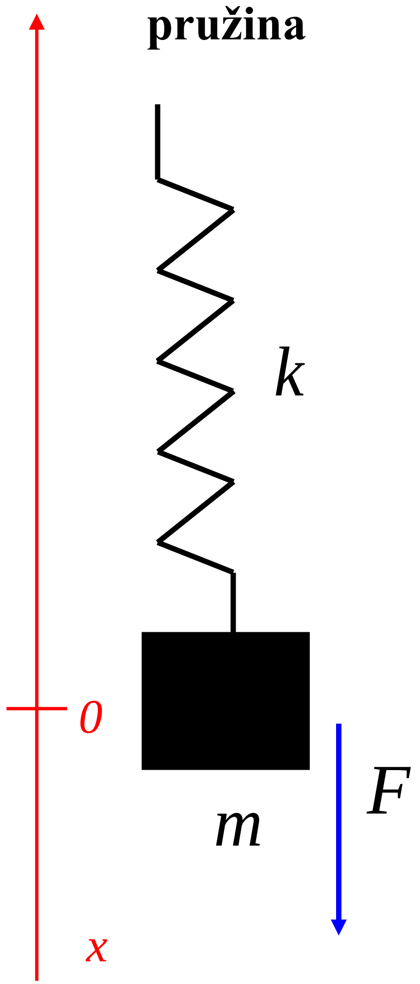
stacionární stav:

$$A_0 = \frac{F_0}{m}\left[\left(\omega_0^2 - \Omega^2\right)^2 + 4\delta^2\Omega^2\right]^{-1/2}$$

$$\vartheta = -\beta \quad \longrightarrow \quad \mathrm{tg}\,\vartheta = \frac{-2\delta\Omega}{\omega_0^2 - \Omega^2}$$

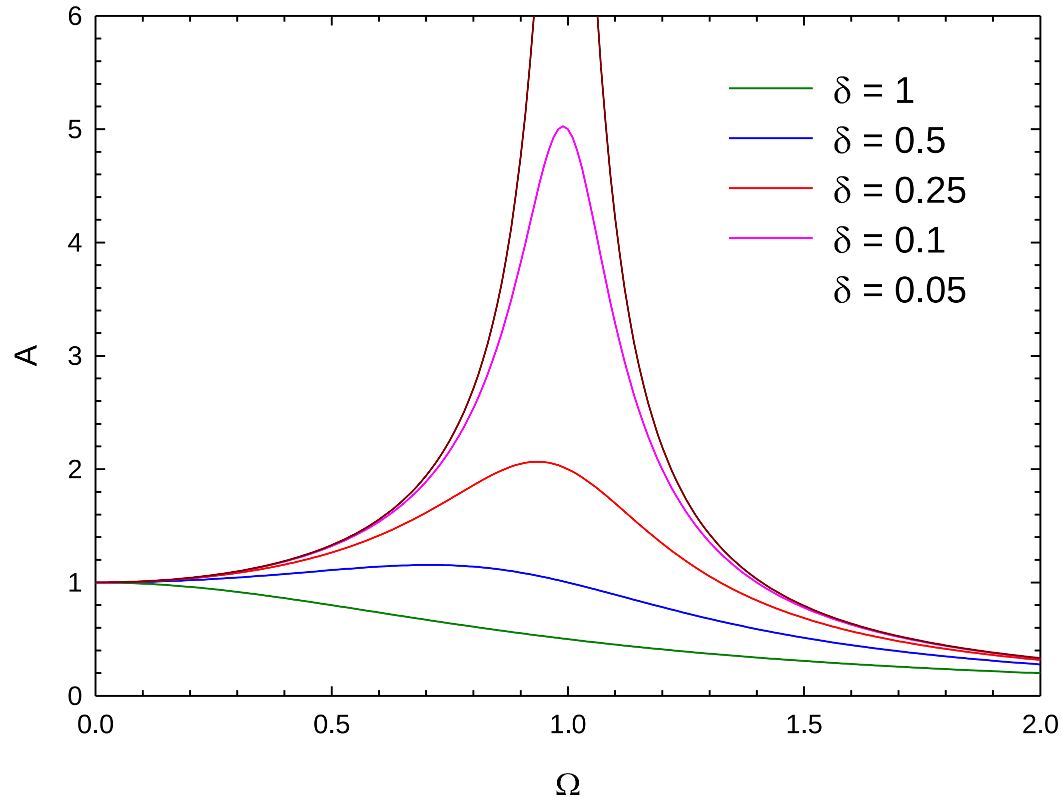
$$x = A_0 e^{i\vartheta} e^{i\Omega t} = A_0 \sin\left(\Omega t + \vartheta\right)$$

Nucené kmity s tlumením



- **amplituda kmitů**

$\omega_0 = 1, F_0 = 1, m = 1$



Nucené kmity s tlumením

• **amplituda kmitů**

$$\frac{\mathrm{d}A_0}{\mathrm{d}\Omega} = \frac{2F_0\left(\omega_0^2 - \Omega^2 - 2\delta^2\right)\Omega}{m\left[\left(\omega_0^2 - \Omega^2\right)^2 + 4\delta^2\Omega^2\right]^{3/2}}$$

$$\frac{\mathrm{d}A_0}{\mathrm{d}\Omega} = 0$$

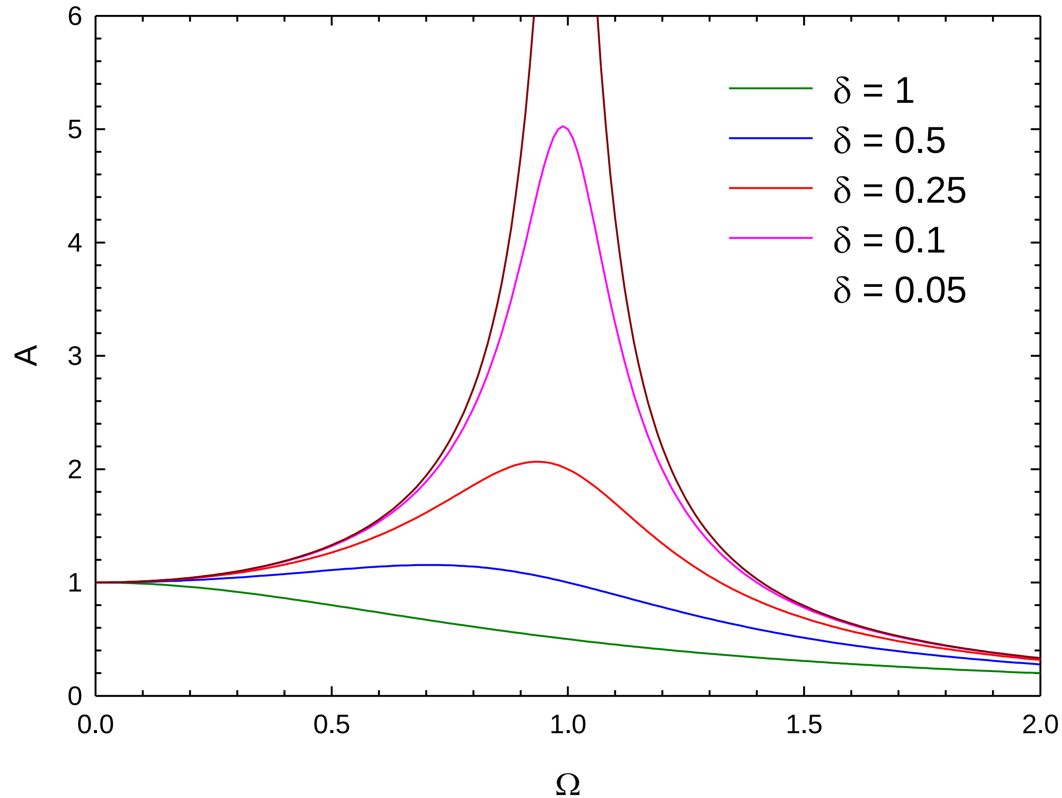
↓

• **rezonance amplitudy**

$$\Omega_r = \sqrt{\omega_0^2 - 2\delta^2}$$

$$A_0(\Omega_r) = \frac{F_0}{2m\delta\left(\omega_0^2 - \delta^2\right)^{1/2}}$$

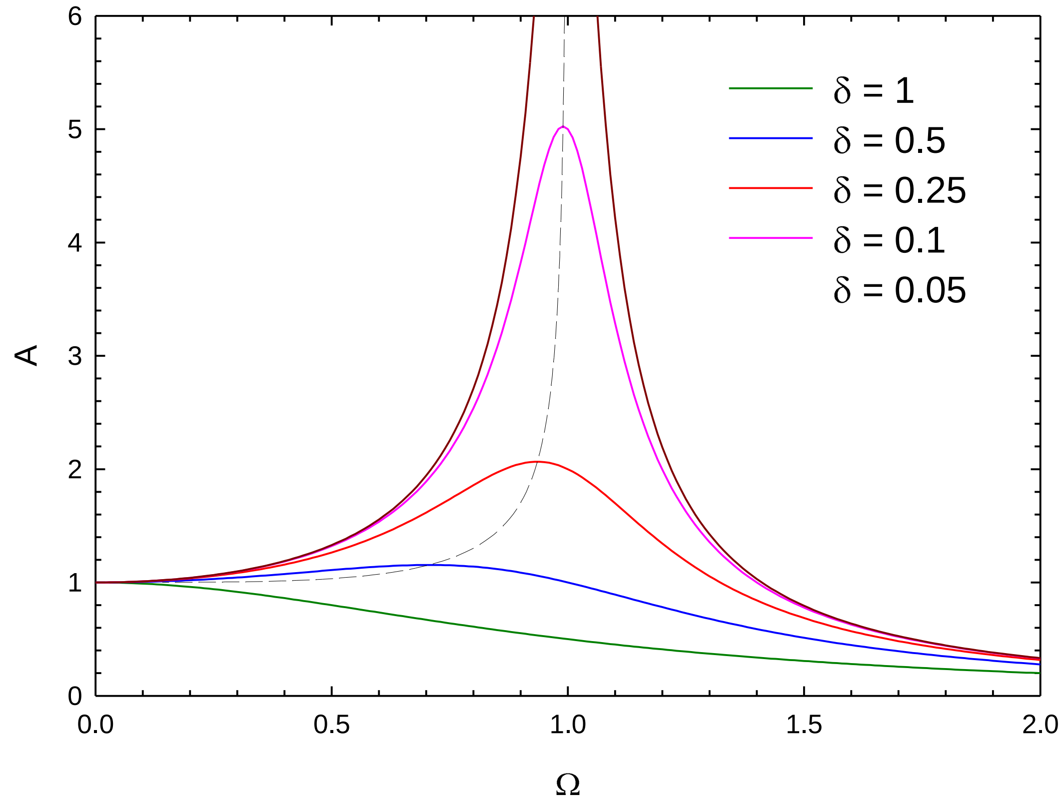
$\omega_0 = 1,\ F_0 = 1,\ m = 1$



Nucené kmity s tlumením

• **amplituda kmitů**

$\omega_0 = 1,\ F_0 = 1,\ m = 1$



$$\frac{dA_0}{d\Omega} = \frac{2F_0\left(\omega_0^2 - \Omega^2 - 2\delta^2\right)\Omega}{m\left[\left(\omega_0^2 - \Omega^2\right)^2 + 4\delta^2\Omega^2\right]^{3/2}}$$

$$\frac{dA_0}{d\Omega} = 0$$

↓

• **rezonance amplitudy**

$$\Omega_r = \sqrt{\omega_0^2 - 2\delta^2}$$

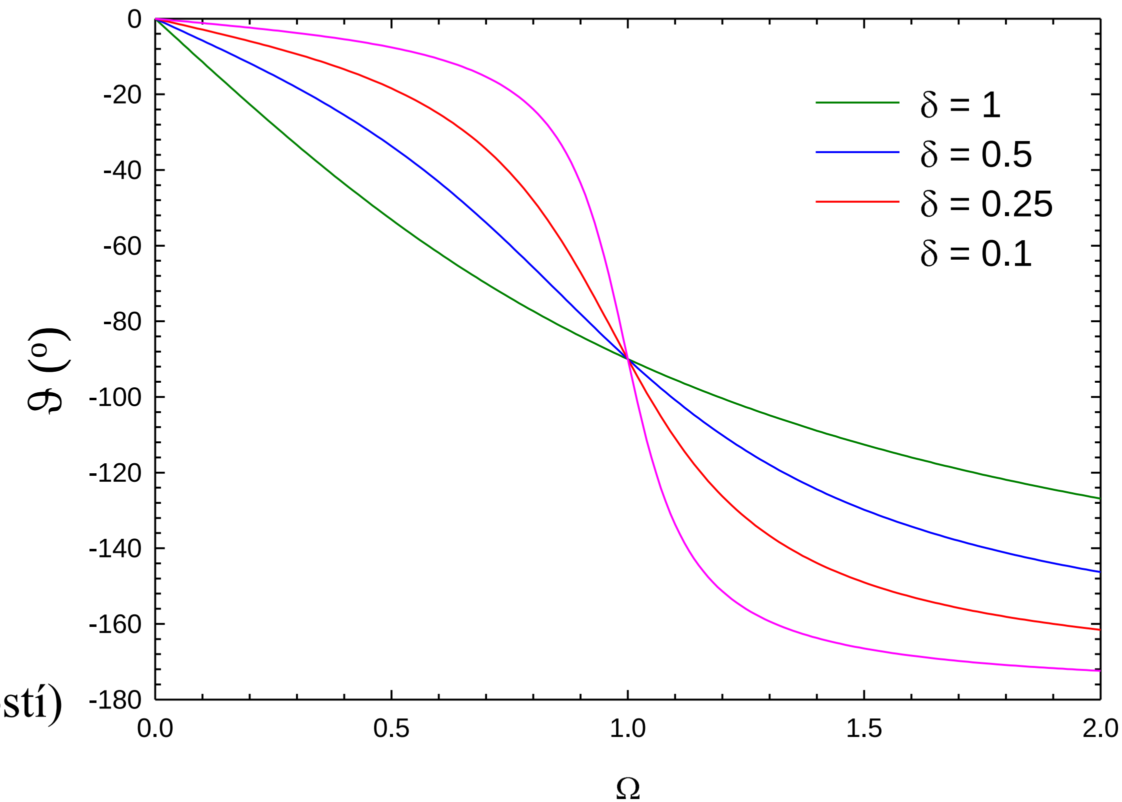
$$A_0(\Omega_r) = \frac{F_0}{2m\delta\left(\omega_0^2 - \delta^2\right)^{1/2}}$$

Nucené kmity s tlumením

$$\mathrm{tg}\,\vartheta = \frac{-2\delta\Omega}{\omega_0^2 - \Omega^2}$$

• **fázový posuv**

$\omega_0 = 1,\ F_0 = 1,\ m = 1$



• malé Ω: pohyb přibližně ve fázi s vynucující silou

• oblast rezonance $\Omega \approx \omega_0$: fázové zpoždění –π/2 (pohyb je přibližně ve fázi s rychlostí)

• velké Ω: fázové zpoždění –π

Nucené kmity s tlumením

• mechanická energie:

$$E = E_k + E_p = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2$$

• z časem klesá:

$$\frac{dE}{dt} = \frac{dx}{dt}\left(m\frac{d^2x}{dt^2} + kx\right) \longrightarrow \frac{dE}{dt} = -h\left(\frac{dx}{dt}\right)^2$$

• pokles energie za periodu *T*:

$$\Delta E = -\int_0^T h\left(\frac{dx}{dt}\right)^2 dt = -\int_0^T hA_0^2\Omega^2\cos^2(\Omega t + \vartheta)\,dt = -hA_0^2\Omega^2\frac{T}{2}$$

$$\frac{h}{m} = 2\delta \quad T = \frac{2\pi}{\Omega} \longrightarrow \Delta E = -2\pi\delta m A_0^2\Omega$$

tuto ztracenou energii musí dodat vynucující síla, tj. tuto práci musí vykonat aby udržela kmity

• výkon vynucující síly: $P_F = \frac{-\Delta E}{T} = \delta m A_0^2 \Omega^2$

Nucené kmity s tlumením

• výkon vynucovací síly:

$$P_F = \delta m A_0^2 \Omega^2 = \frac{F_0^2 \Omega^2 \delta}{m\left[(\omega_0^2 - \Omega^2)^2 + 4\delta^2\Omega^2\right]}$$

$$\frac{\mathrm{d}P_F}{\mathrm{d}\Omega} = 0 \longrightarrow$$

rezonance výkonu nastává pro

$$\Omega = \omega_0$$

• průměrná mechanická energie vynuceného harmonického kmitu:

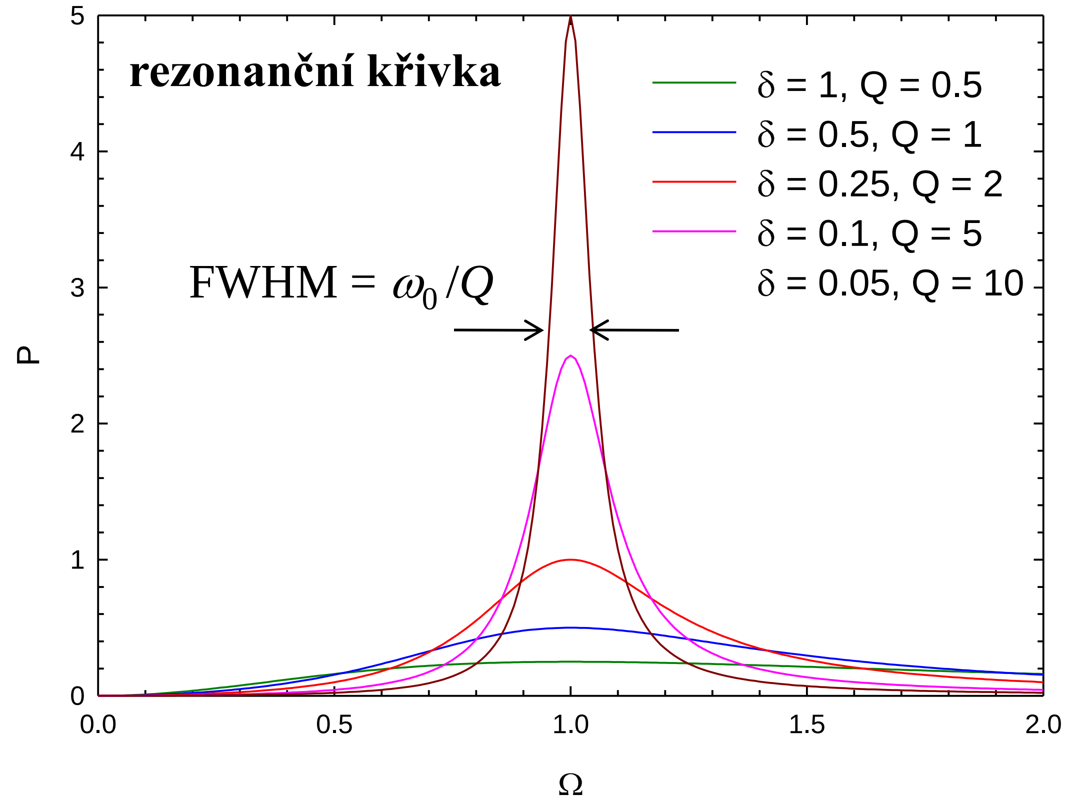
$$\overline{E} = \frac{1}{2} m A_0^2 \Omega^2$$

• činitel jakosti

$$Q = \frac{2\pi \overline{E}}{|\Delta E|} = \frac{2\pi \overline{E}}{P_F T} = \frac{\omega_0}{2\delta}$$

$$\Delta E = 2\pi \delta m A_0^2 \Omega$$

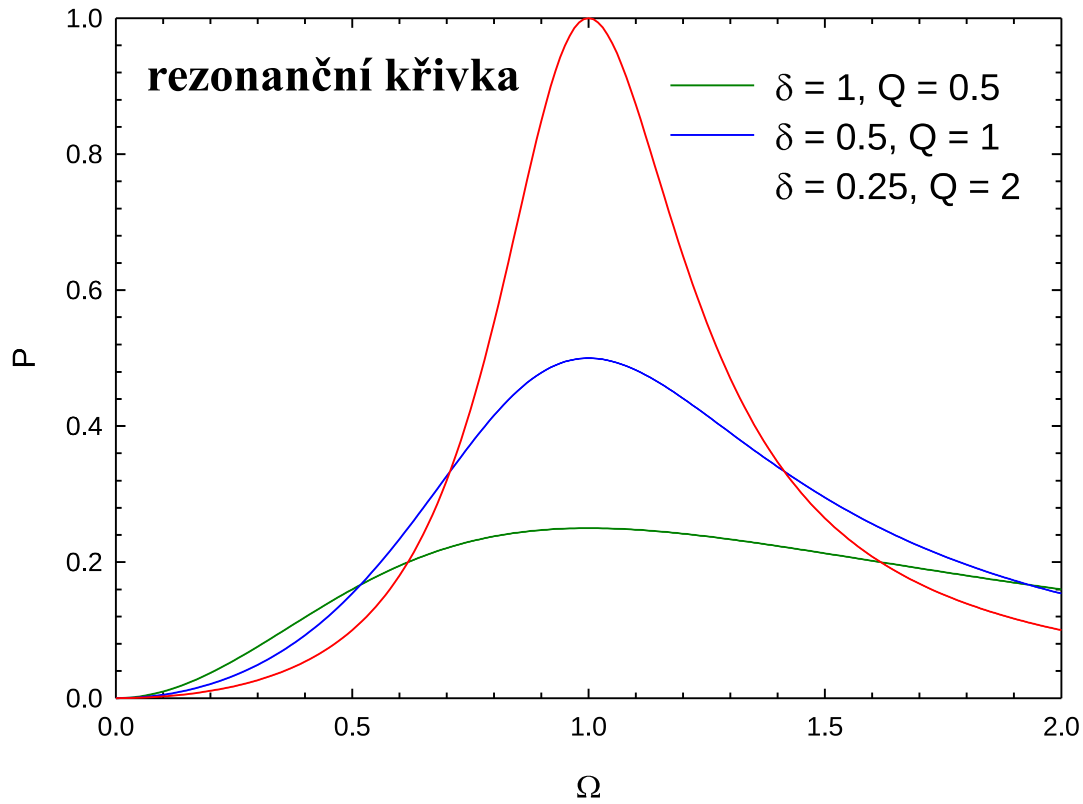
při rezonanci $\Omega = \omega_0$



Nucené kmity s tlumením

• výkon vynucovací síly:

$$P_F = \delta m A_0^2 \Omega^2 = \frac{F_0^2 \Omega^2 \delta}{m\left[(\omega_0^2 - \Omega^2)^2 + 4\delta^2\Omega^2\right]}$$



Nucené kmity s tlumením

- výkon vynucovací síly:

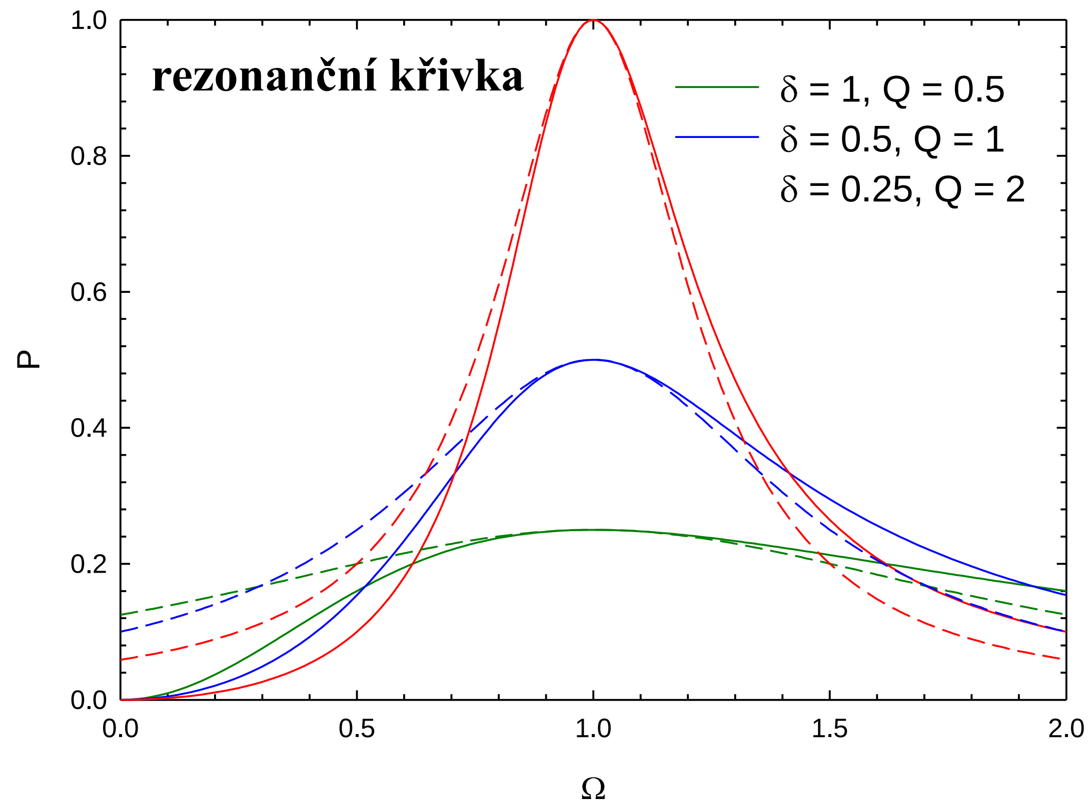
$$P_F = \delta m A_0^2 \Omega^2 = \frac{F_0^2 \Omega^2 \delta}{m\left[(\omega_0^2 - \Omega^2)^2 + 4\delta^2\Omega^2\right]}$$

- v blízkosti rezonance $\Omega \approx \omega_0$:

Lorentzián

$$P_F \approx \frac{F_0^2 \delta}{4m\left[(\omega_0 - \Omega)^2 + \delta^2\right]}$$

$$\mathrm{FWHM} = 2\delta = \frac{\omega_0}{Q}$$

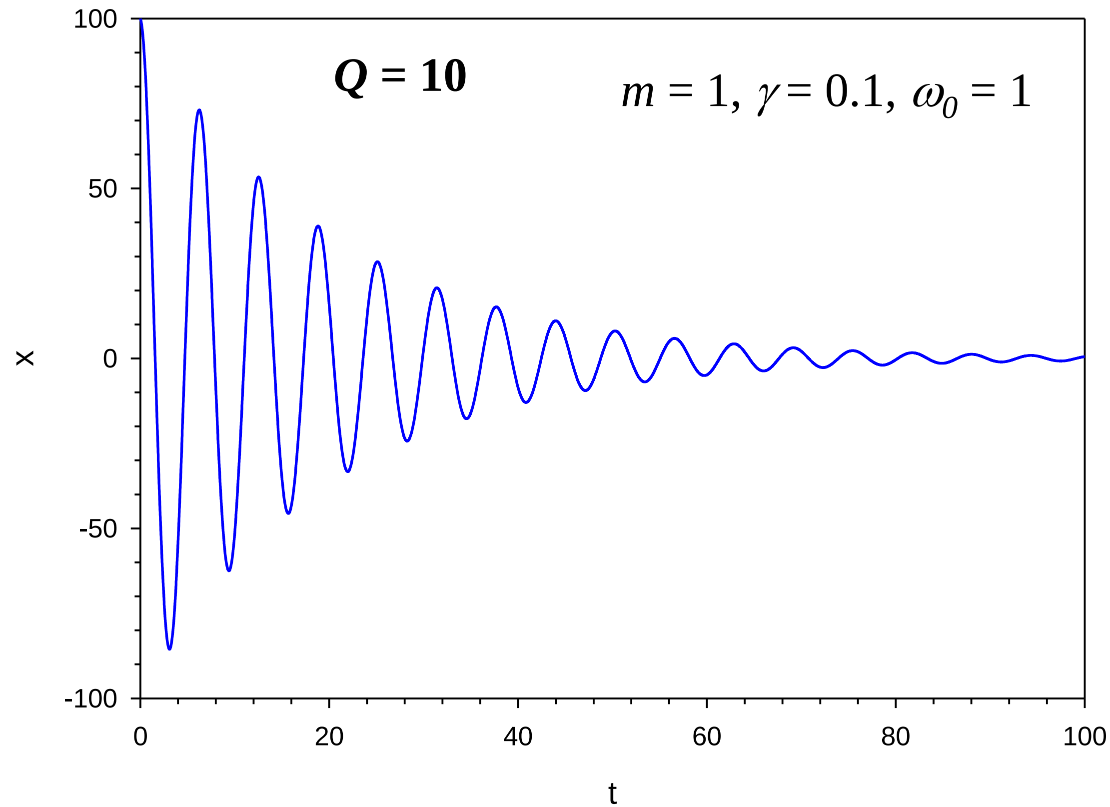


Nucené kmity s tlumením

- pokud přestane působit vynucovací síla bude amplituda kmitů klesat jako $e^{-\delta t} = e^{-\frac{\omega_0}{2Q}t}$

- za jednu periodu poklesne faktorem $e^{-\frac{\omega_0}{2Q}T} = e^{-\frac{\pi}{Q}}$

Q – za kolik cyklů se amplituda zmenší faktorem $e^{-\pi}$



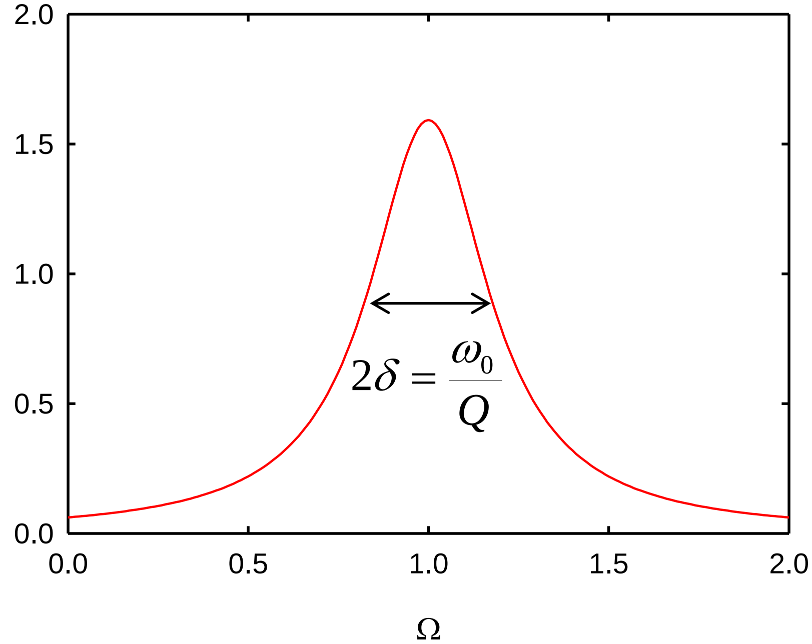
Univerzální rezonanční křivka

- **Lorentzián**

$$I(\Omega) = \frac{\delta}{(\omega_0 - \Omega)^2 + \delta^2}$$

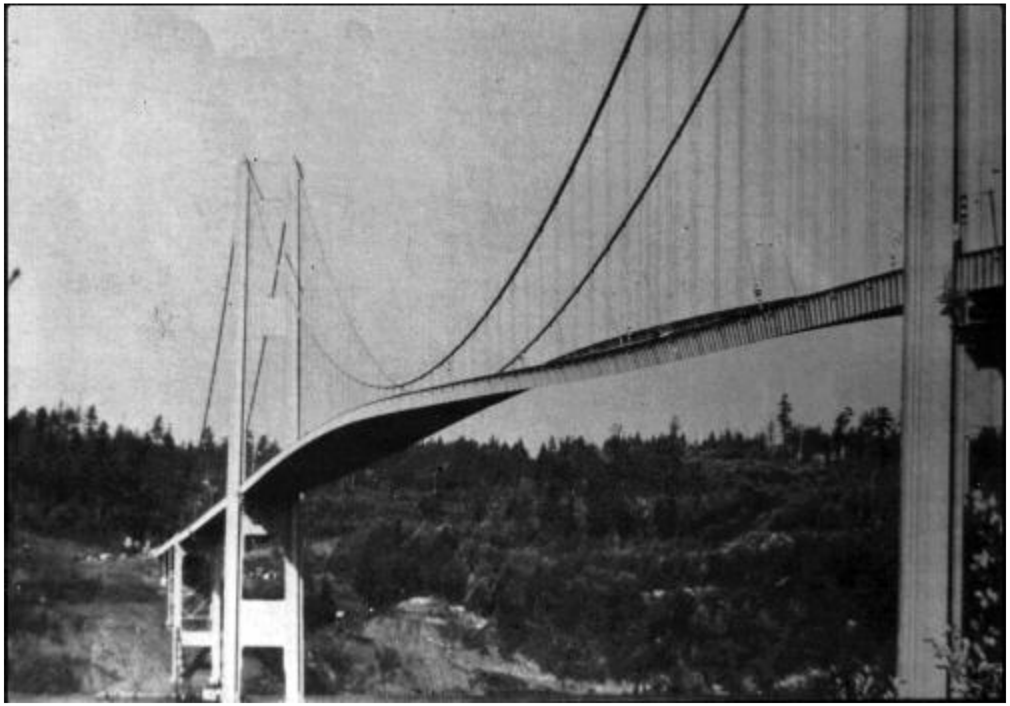
rezonanční frekvence

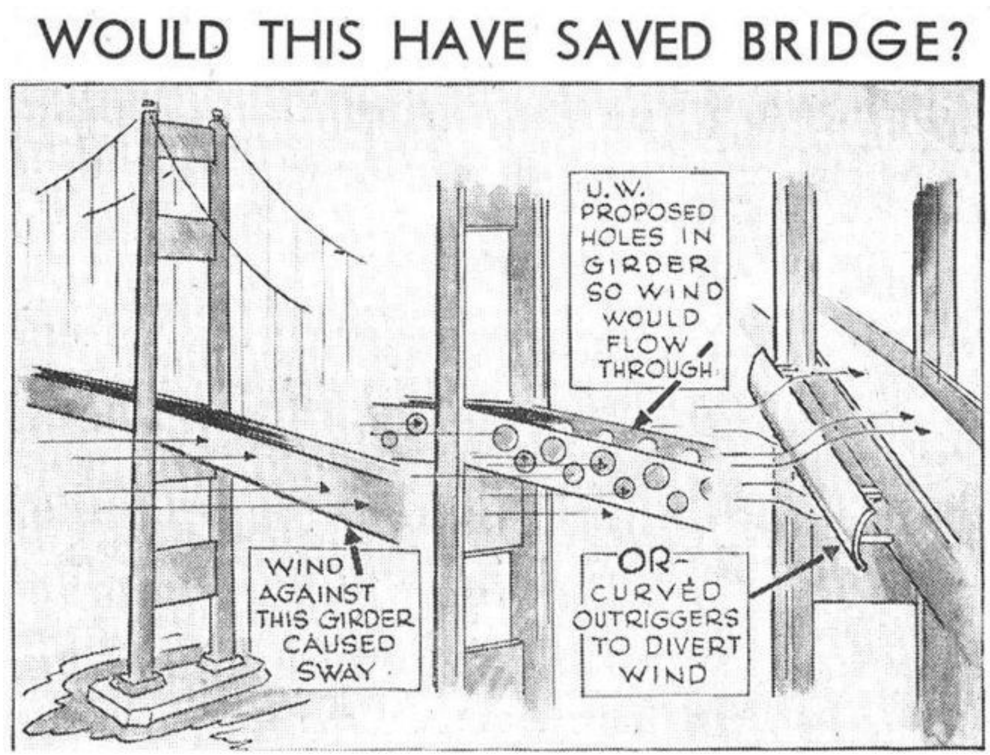
tlumení



Mechanická rezonance

- Tacoma Narrows Bridge (1940), Tacoma, Washington U.S.





Rezonance v elektrických obvodech

- RLC obvod

$$U(t) = U_C + U_R + U_L$$

- kondenzátor $U_C = \dfrac{q}{C}$

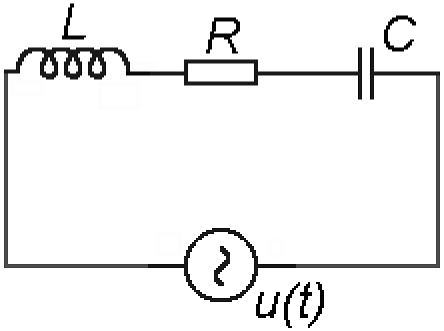
$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = U(t)$$

- odpor $U_R = RI = R\dfrac{dq}{dt}$

$$q = \hat{q}e^{i\Omega t}$$

$$L(i\Omega)^2\hat{q} + Ri\Omega\hat{q} + \frac{1}{C}\hat{q} = \hat{U}$$

- cívka $U_L = L\dfrac{dI}{dt} = L\dfrac{d^2q}{dt^2}$



$$\hat{q} = \frac{\hat{U}}{-L\Omega^2 + iR\Omega + \dfrac{1}{C}} = \frac{\hat{U}}{L\left(\omega_0^2 - \Omega^2 + i2\delta\Omega\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad 2\delta = \frac{R}{L}$$

Ekvivalence mechanické a elektrické rezonance

• **vlastnost**	• **mechanická rezonance**	• **elektrická rezonance**
• nezávisle proměnná	• čas t	• čas t
• závisle proměnná	• poloha x	• náboj q
• setrvačnost	• hmotnost m	• indukčnost L
• odpor	• koeficient tření, $h = 2\delta m$	• elektrický odpor, $R = 2\delta L$
• tuhost	• mechanická tuhost k	• 1 / kapacita, C^{-1}
• vlastní frekvence	• $\omega_0^2 = \frac{k}{m}$	• $\omega_0^2 = \frac{1}{LC}$
• perioda	• $T = 2\pi\sqrt{\frac{m}{k}}$	• $T = 2\pi\sqrt{LC}$
• činitel jakosti	• $Q = \frac{\omega_0}{2\delta}$	• $Q = \frac{\omega_0 L}{R}$

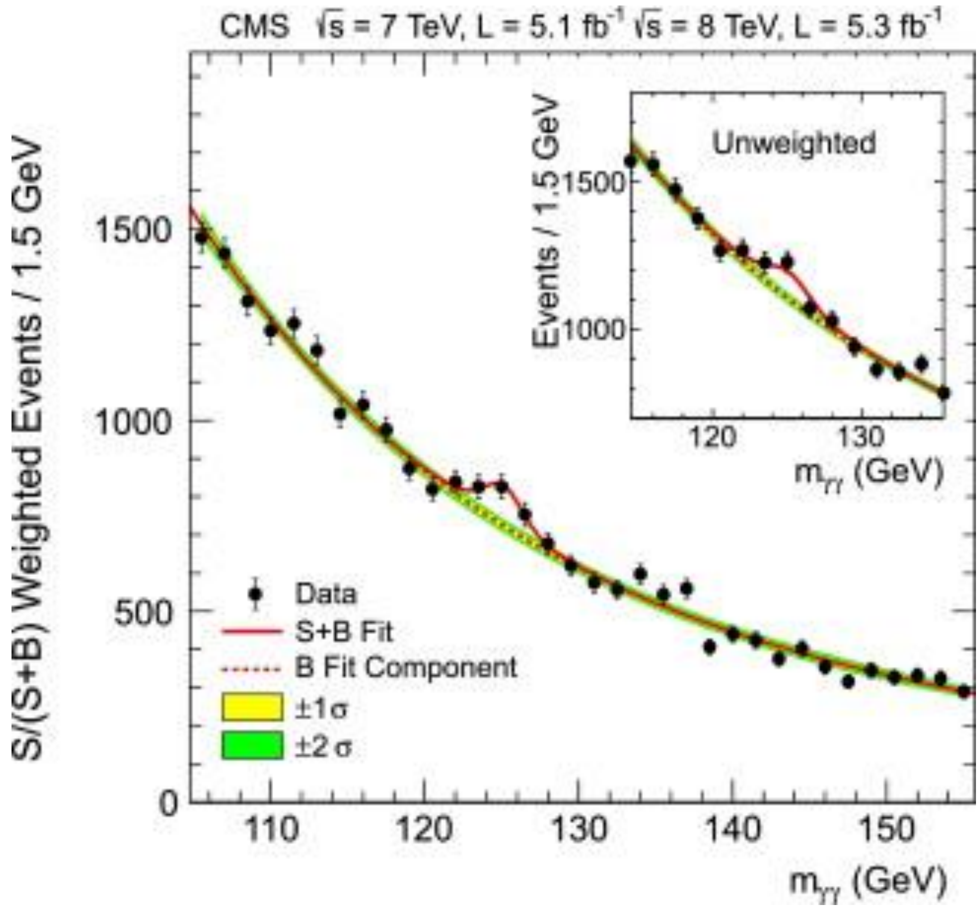
Higgsův boson

$$I(\Omega) = \frac{\delta}{(\omega_0 - \Omega)^2 + \delta^2}$$

klidová hmotnost (ω_0)

rychlost rozpadu (δ)

ω_0 = 125 GeV



Skládání kmitů

$$x_1 = \hat{A}_1 e^{i\omega t} = A_{0_1} e^{i\varphi_1} e^{i\omega t}$$

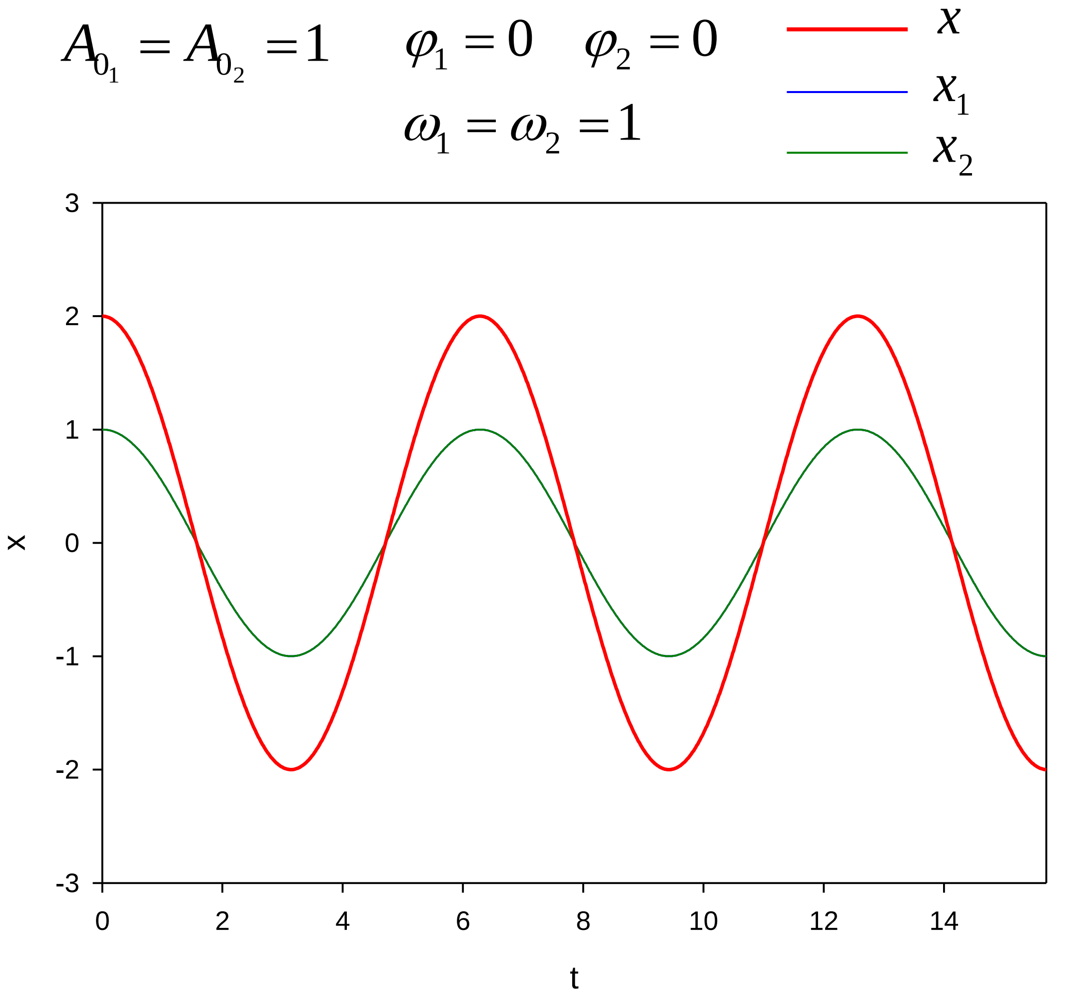
$$x_2 = \hat{A}_2 e^{i\omega t} = A_{0_2} e^{i\varphi_2} e^{i\omega t}$$

$$x = x_1 + x_2 = \left(A_{0_1} e^{i\varphi_1} + A_{0_2} e^{i\varphi_2}\right) e^{i\omega t} = \hat{x} e^{i\omega t}$$

$$\hat{x}^2 = A_{0_1}^2 + A_{0_2}^2 + 2A_{0_1} A_{0_2} \cos(\varphi_2 - \varphi_1)$$

$A_{0_1} = A_{0_2} = 1 \quad \varphi_1 = 0 \quad \varphi_2 = 0$

$\omega_1 = \omega_2 = 1$



Skládání kmitů

$$x_1 = \hat{A}_1 e^{i\omega t} = A_{0_1} e^{i\varphi_1} e^{i\omega t}$$

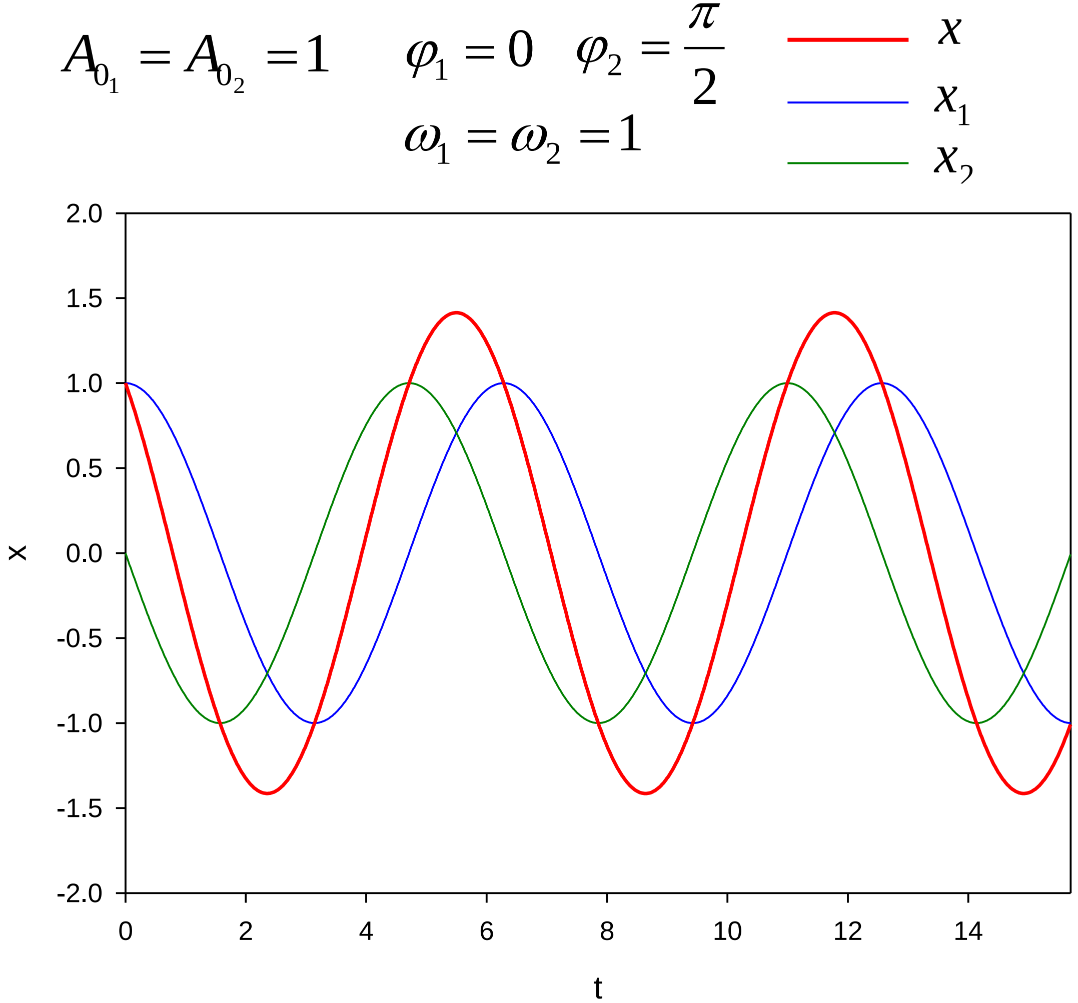
$$x_2 = \hat{A}_2 e^{i\omega t} = A_{0_2} e^{i\varphi_2} e^{i\omega t}$$

$$x = x_1 + x_2 = \left(A_{0_1} e^{i\varphi_1} + A_{0_2} e^{i\varphi_2}\right) e^{i\omega t} = \hat{x} e^{i\omega t}$$

$$\hat{x}^2 = A_{0_1}^2 + A_{0_2}^2 + 2A_{0_1} A_{0_2} \cos(\varphi_2 - \varphi_1)$$

$A_{0_1} = A_{0_2} = 1 \quad \varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$

$\omega_1 = \omega_2 = 1$



Skládání kmitů

$$x_1 = \hat{A}_1 e^{i\omega t} = A_{0_1} e^{i\varphi_1} e^{i\omega t}$$

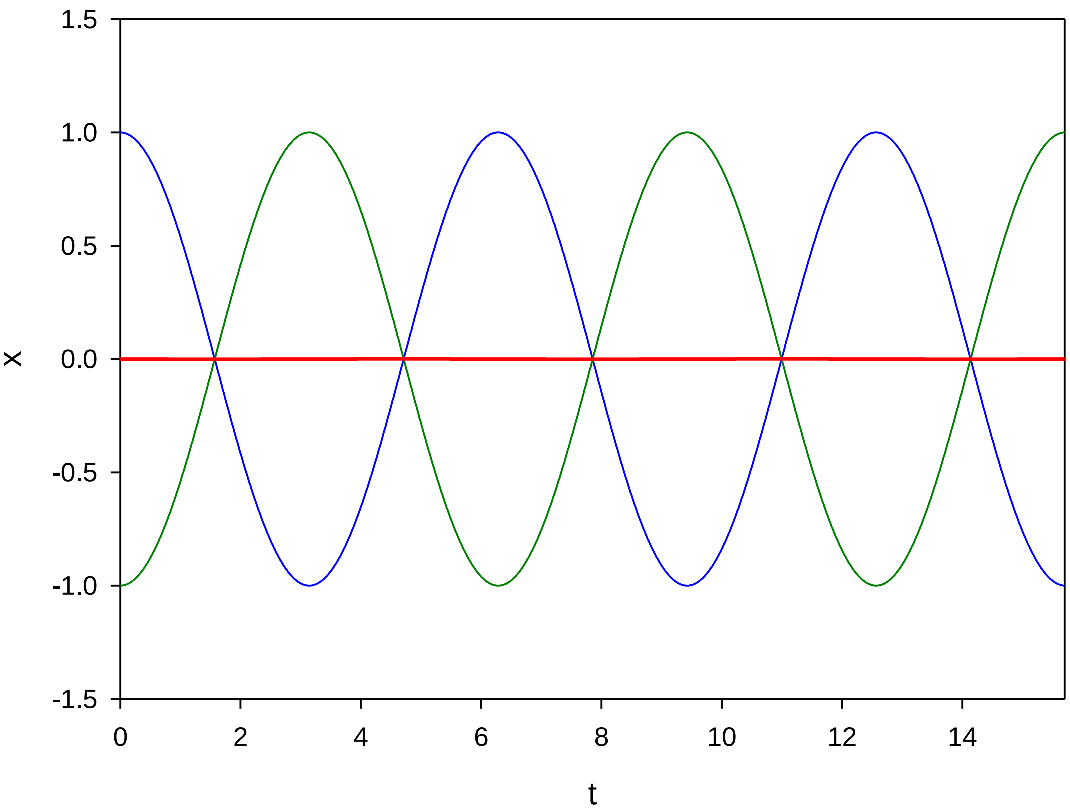
$$x_2 = \hat{A}_2 e^{i\omega t} = A_{0_2} e^{i\varphi_2} e^{i\omega t}$$

$$x = x_1 + x_2 = \left(A_{0_1} e^{i\varphi_1} + A_{0_2} e^{i\varphi_2}\right) e^{i\omega t} = \hat{x} e^{i\omega t}$$

$$\hat{x}^2 = A_{0_1}^2 + A_{0_2}^2 + 2 A_{0_1} A_{0_2} \cos(\varphi_2 - \varphi_1)$$

$A_{0_1} = A_{0_2} = 1 \quad \varphi_1 = 0 \quad \varphi_2 = \pi$

$\omega_1 = \omega_2 = 1$



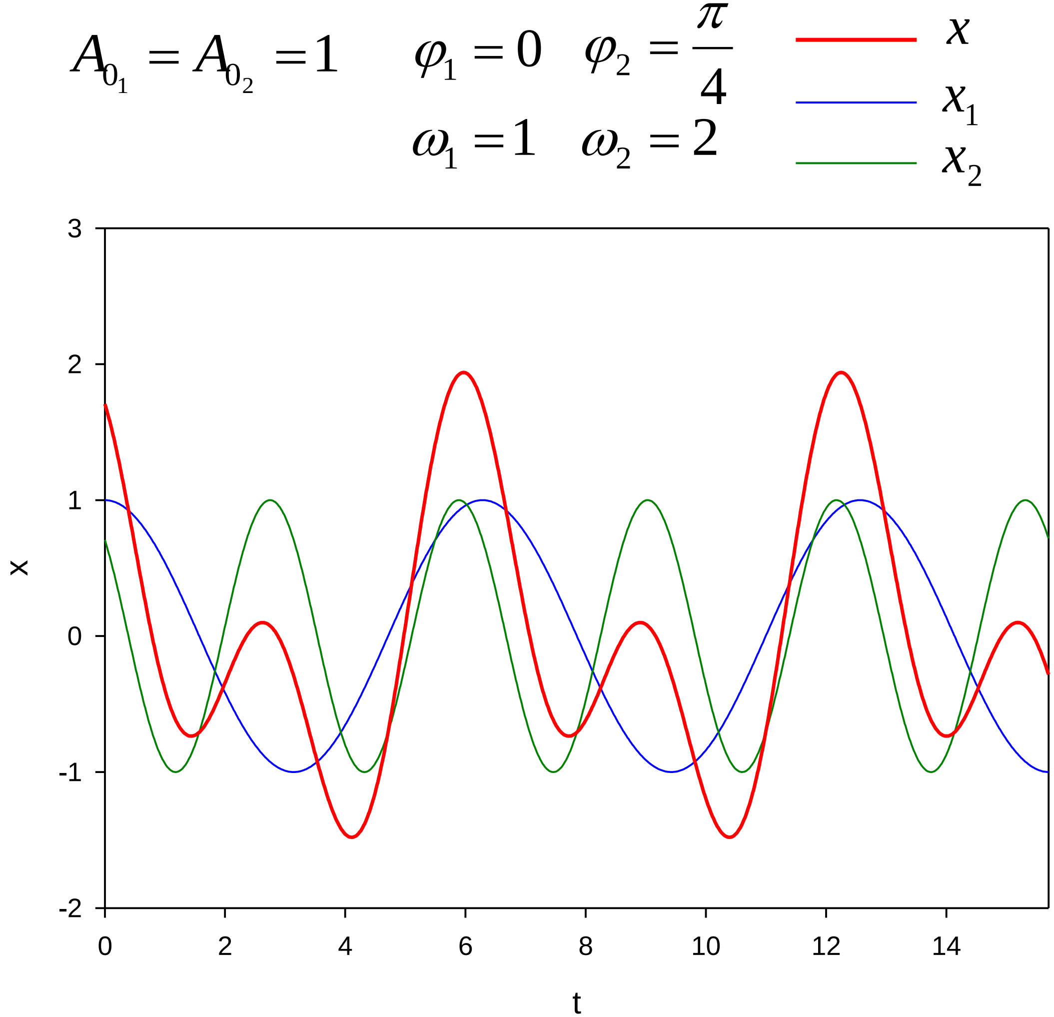
Skládání kmitů

$$x_1 = \hat{A}_1 e^{i\omega_1 t} = A_{0_1} e^{i\varphi_1} e^{i\omega_1 t}$$

$$x_2 = \hat{A}_2 e^{i\omega_2 t} = A_{0_2} e^{i\varphi_2} e^{i\omega_2 t}$$

$$A_{0_1} = A_{0_2} = 1 \quad \varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{4}$$

$$\omega_1 = 1 \quad \omega_2 = 2$$



Skládání kmitů - rázy

$$x_1 = \hat{A}e^{i\omega_1 t} = A_0 e^{i\omega_1 t}$$

$$\omega_1 \approx \omega_2 \equiv \omega$$

$$x_2 = \hat{A}e^{i\omega_2 t} = A_0 e^{i\omega_2 t}$$

$$\varepsilon \equiv \frac{\omega_2 - \omega_1}{2} \qquad \omega \equiv \frac{\omega_1 + \omega_2}{2}$$

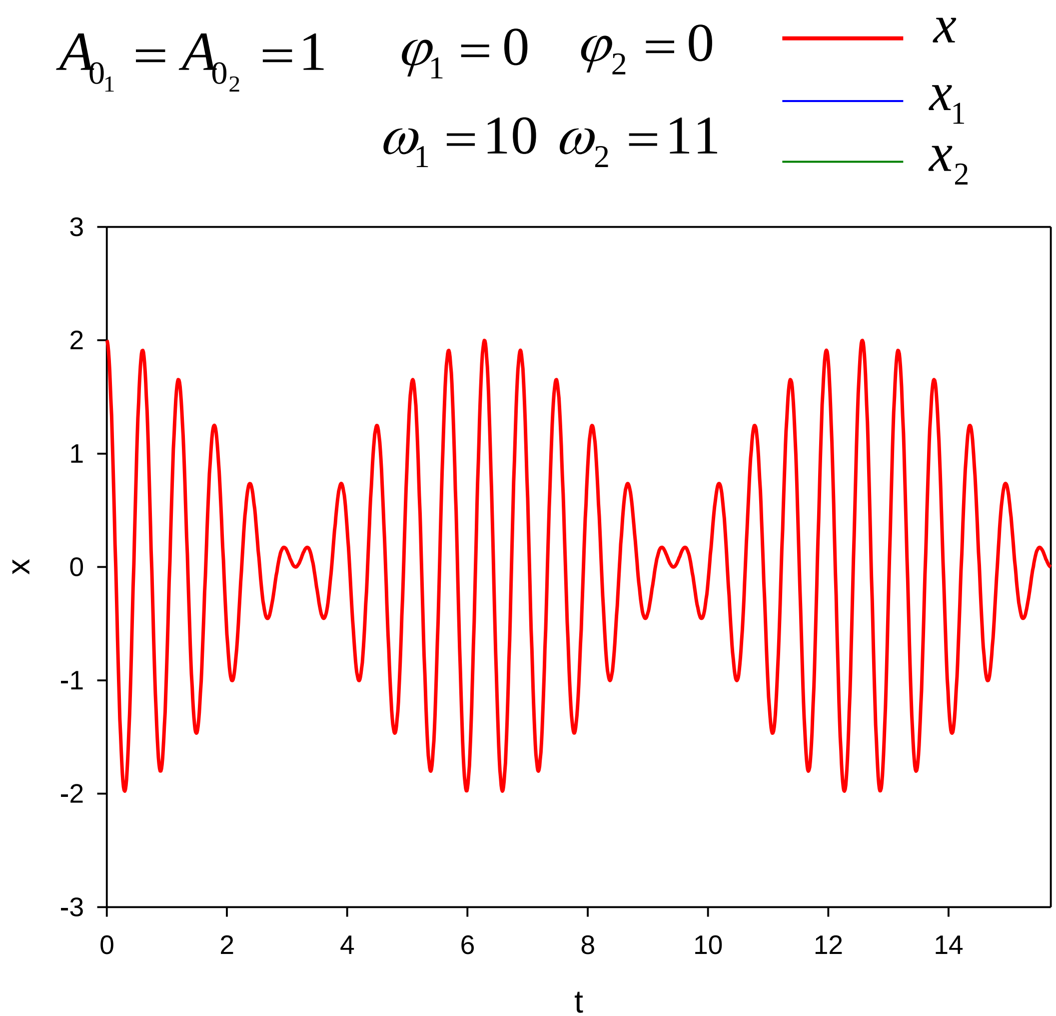
$$\omega_1 \equiv \omega - \varepsilon \qquad \omega_2 \equiv \omega + \varepsilon$$

$$x = x_1 + x_2 = A_0\left(e^{i\varepsilon t} + e^{-i\varepsilon t}\right)e^{i\omega t}$$

$$x = 2A_0 \cos\left(\frac{\omega_2 - \omega_1}{2}t\right)\cos\left(\frac{\omega_1 + \omega_2}{2}t\right)$$

$$A_{0_1} = A_{0_2} = 1 \quad \varphi_1 = 0 \quad \varphi_2 = 0$$

$$\omega_1 = 10 \quad \omega_2 = 11$$



Skládání kolmých kmitů

$$x = \hat{A}_x e^{i\omega_1 t} = A_{0x} e^{i\varphi_1} e^{i\omega_1 t}$$

$$y = \hat{A}_y e^{i\omega_2 t} = A_{0y} e^{i\varphi_2} e^{i\omega_2 t}$$

```
import numpy as np
import matplotlib.pyplot as plt

omega1=12
phi1=0
omega2=11
phi2=np.pi/2
A1=1
A2=1
dt=0.01
t=np.arange(0,20,dt)
x1=A1*np.sin(omega1*t+phi1)
x2=A2*np.sin(omega2*t+phi2)

fig,ax=plt.subplots(figsize=(6,6))
plt.plot(x1,x2)
ax.set_xlabel('x',fontsize=14)
ax.set_ylabel('y',fontsize=14)
```

Skládání kolmých kmitů

$$x = \hat{A}e^{i\omega_1 t} = A_{0x}e^{i\varphi_1}e^{i\omega_1 t}$$

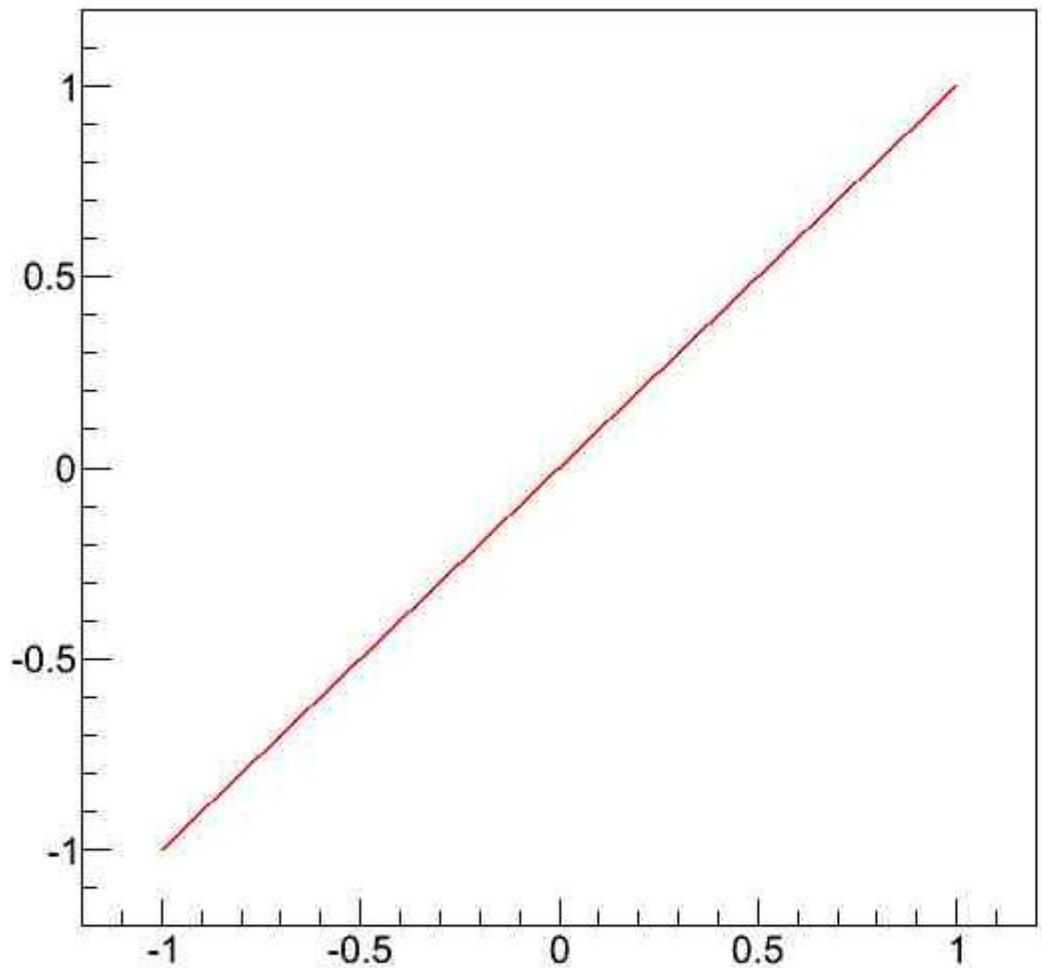
$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = 0$$

$$\omega_1 = 1 \quad \omega_2 = 1$$

trajektorie



Skládání kolmých kmitů

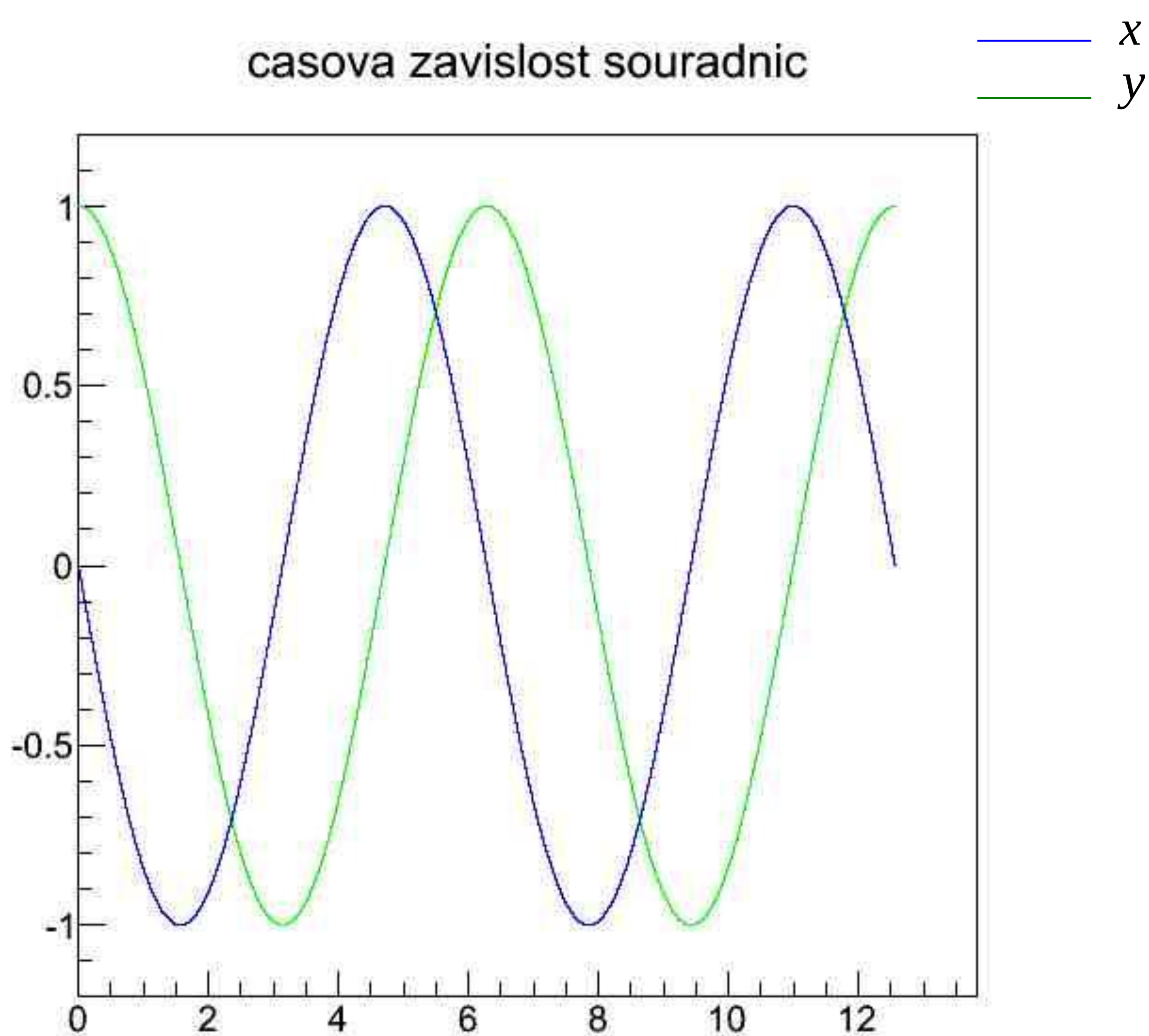
$$x = \hat{A}e^{i\omega_1 t} = A_{0x}e^{i\varphi_1}e^{i\omega_1 t}$$

$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 1 \quad \omega_2 = 1$$



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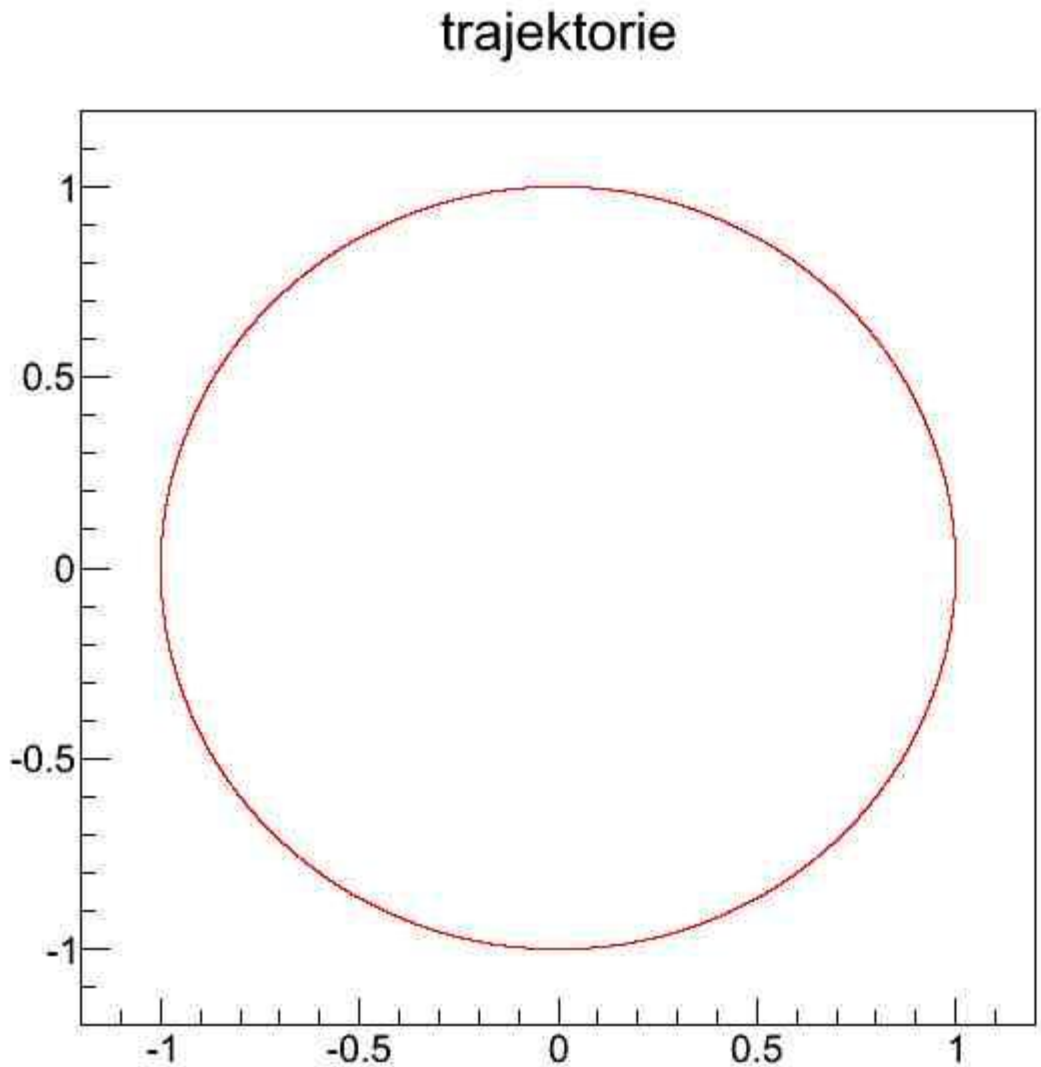
$$\omega_1 = 1 \quad \omega_2 = 1$$



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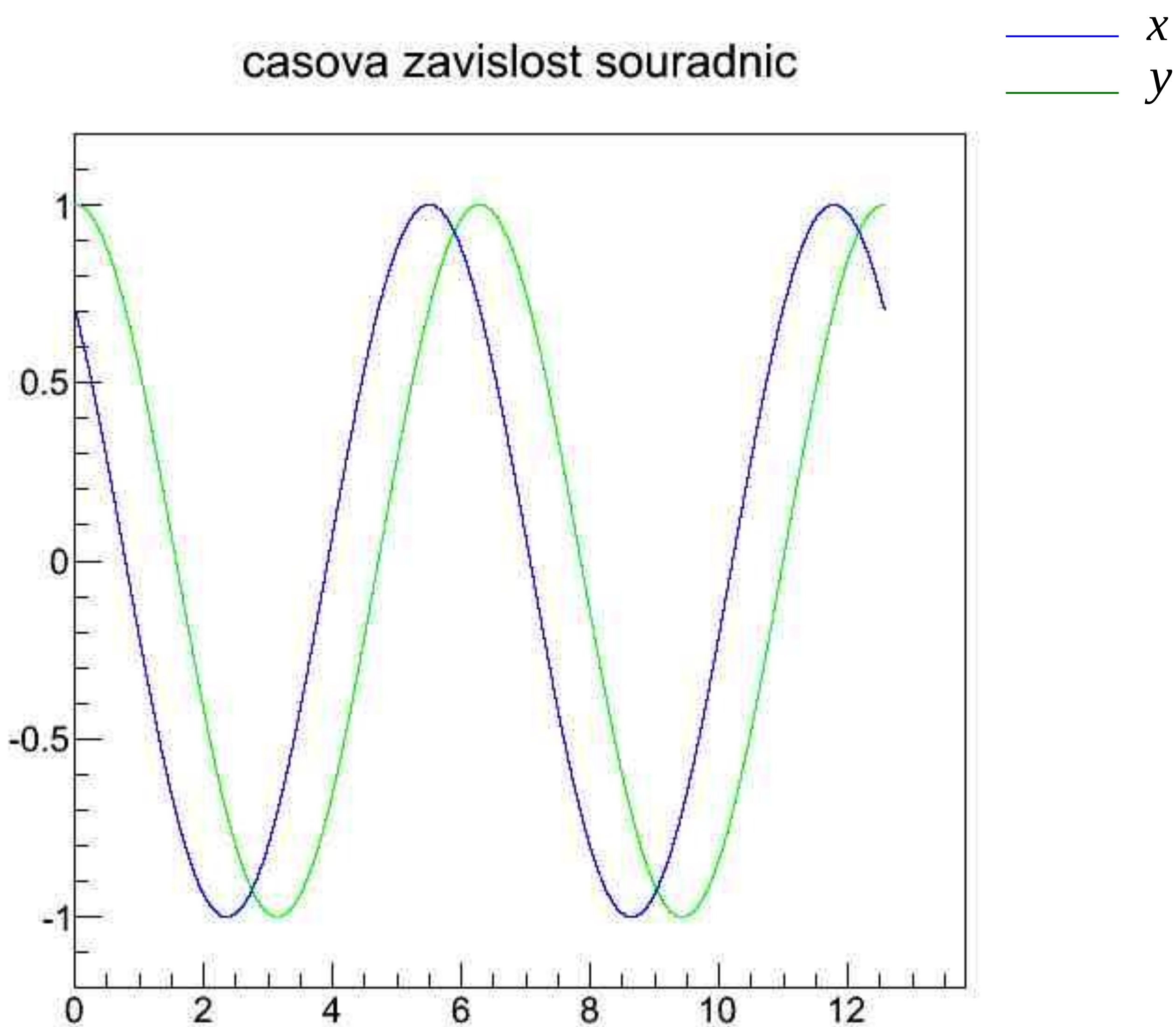
$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{4}$$

$$\omega_1 = 1 \quad \omega_2 = 1$$



Skládání kolmých kmitů

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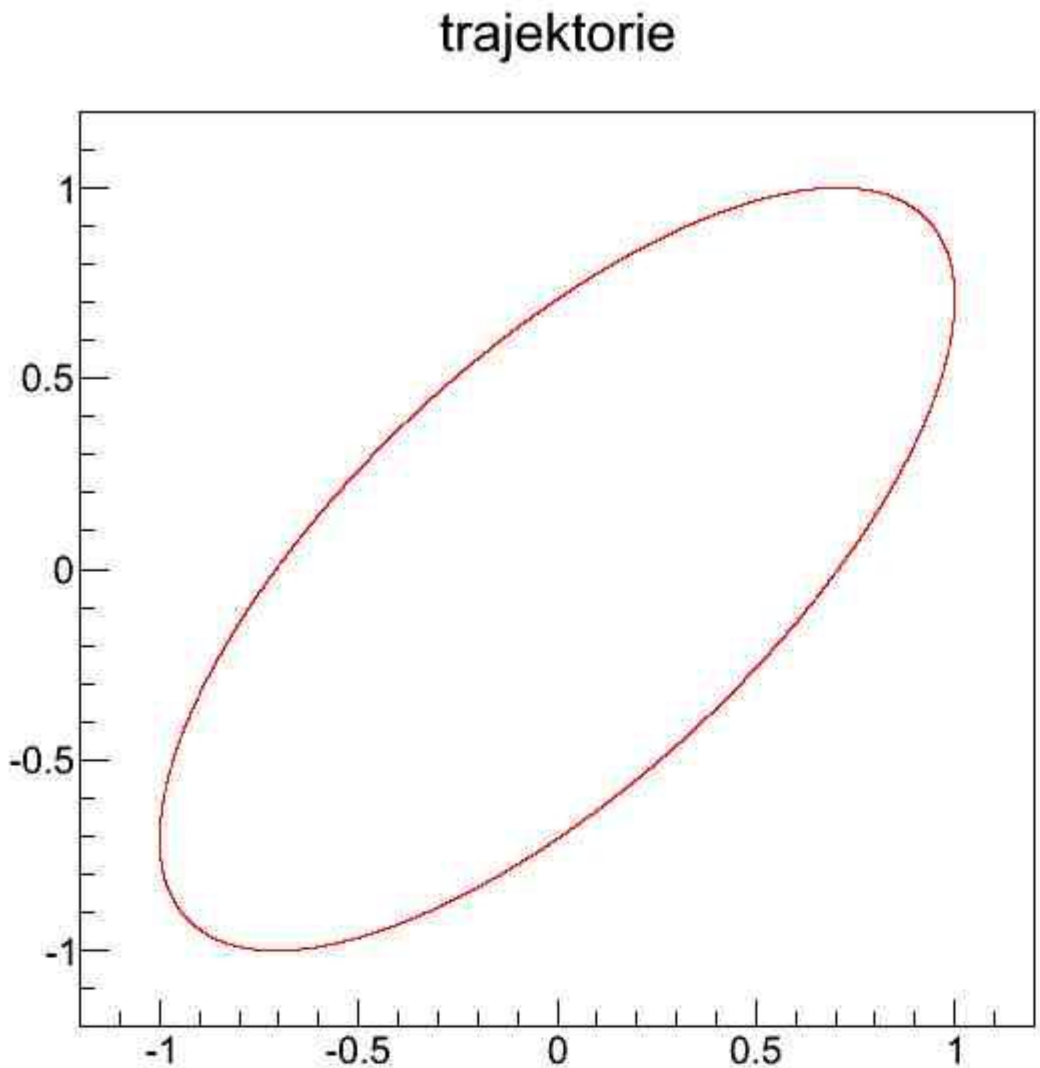
$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{4}$$

$$\omega_1 = 1 \quad \omega_2 = 1$$



Skládání kolmých kmitů

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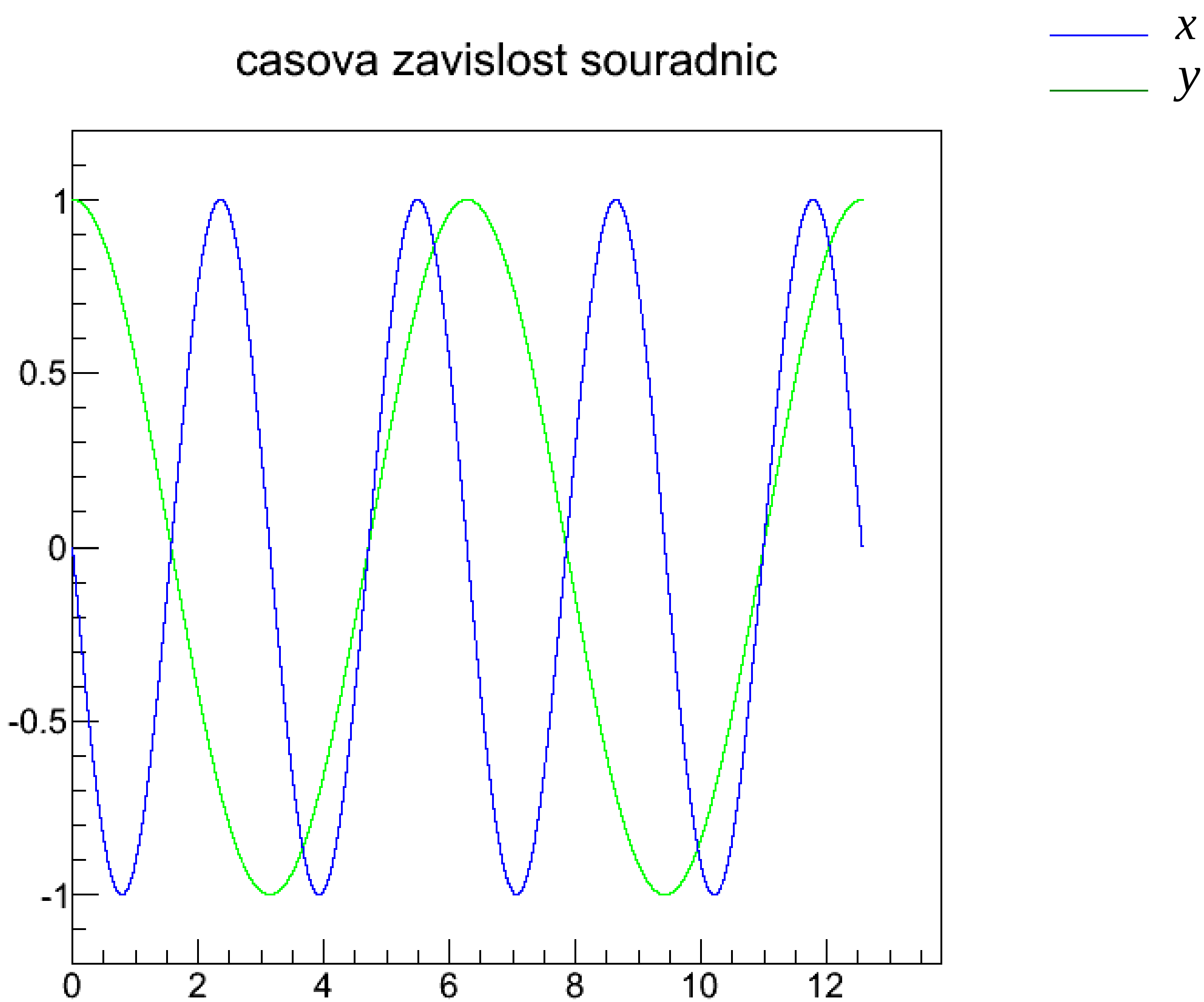
$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 2 \quad \omega_2 = 1$$



Skládání kolmých kmitů

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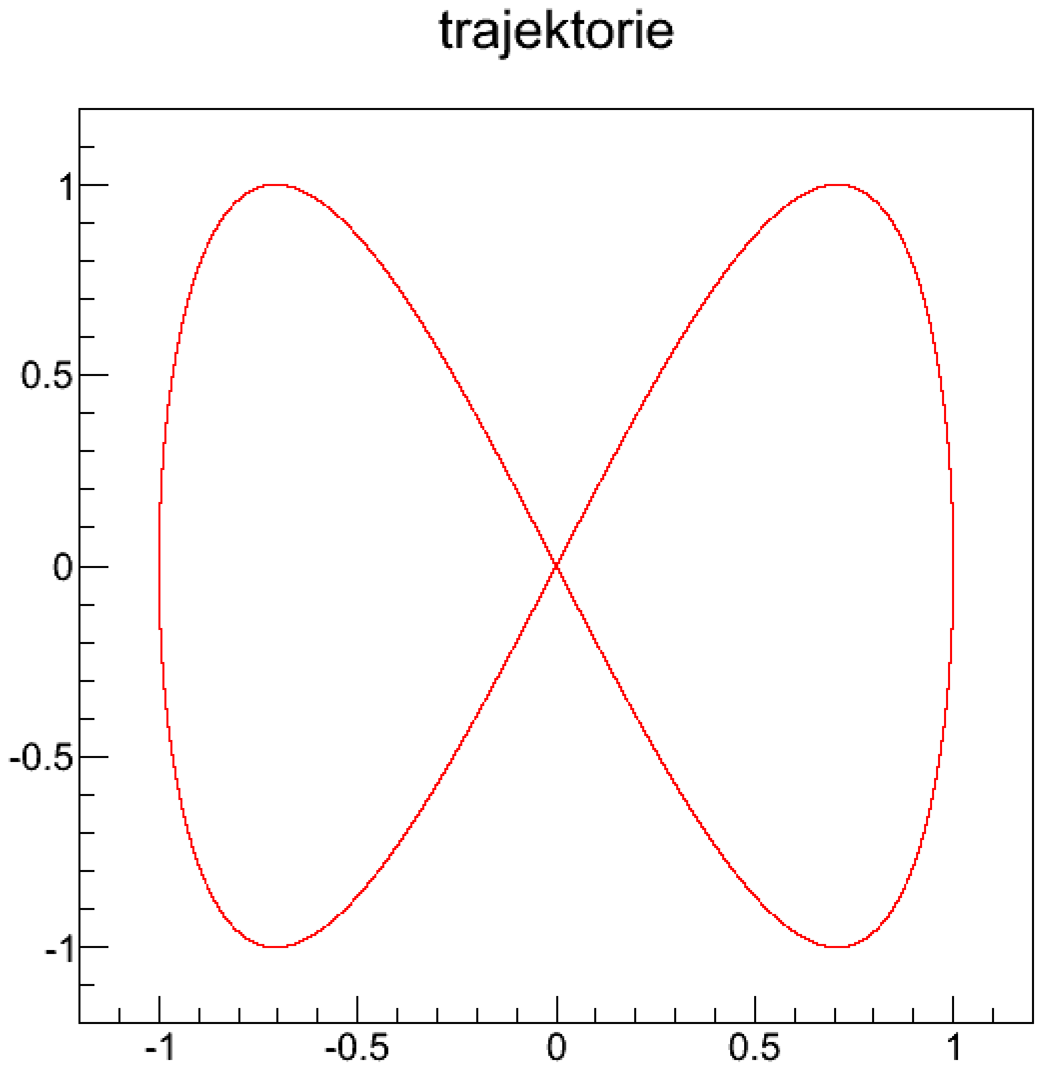
$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 2 \quad \omega_2 = 1$$



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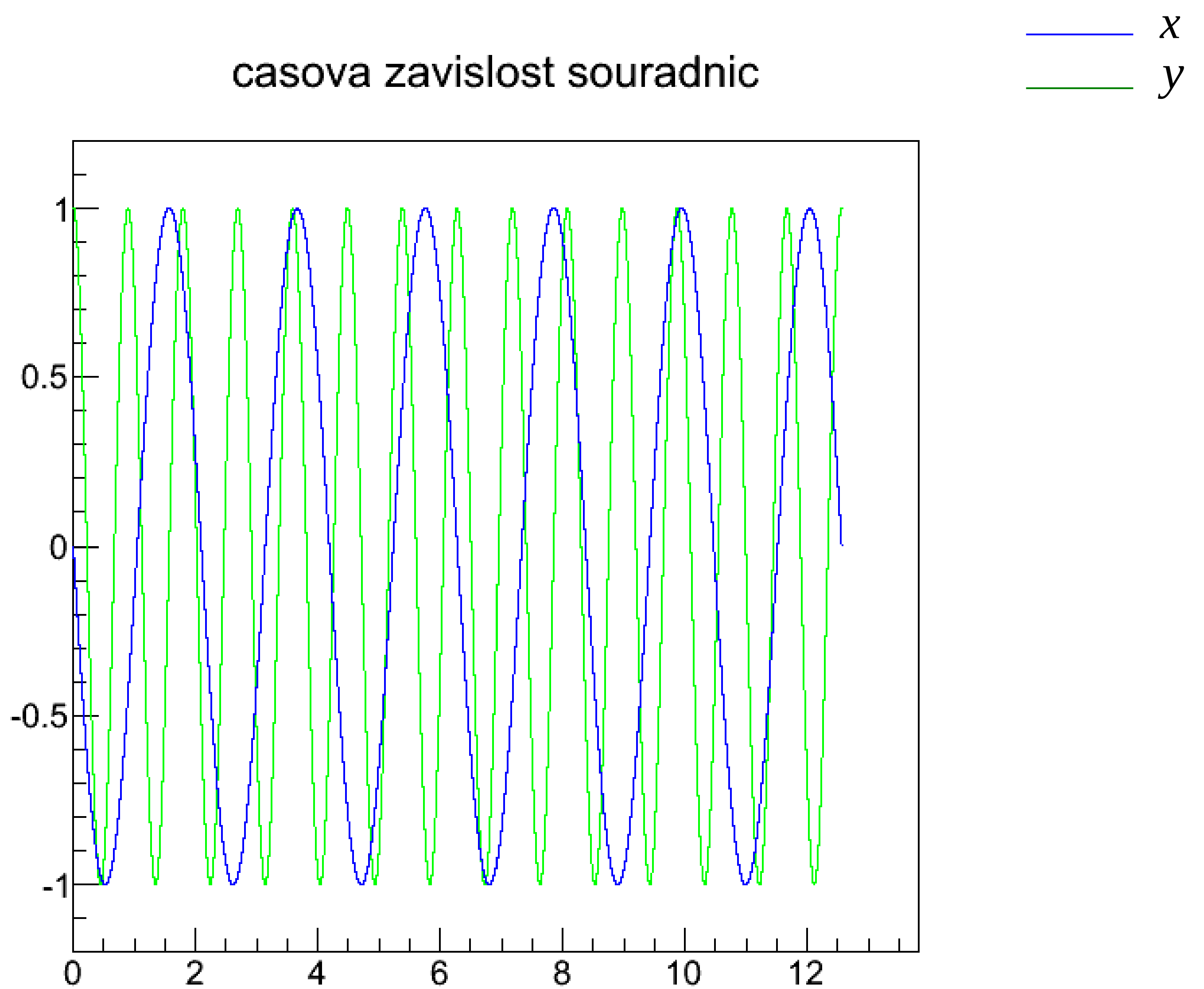
$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 3 \quad \omega_2 = 7$$



Skládání kolmých kmitů

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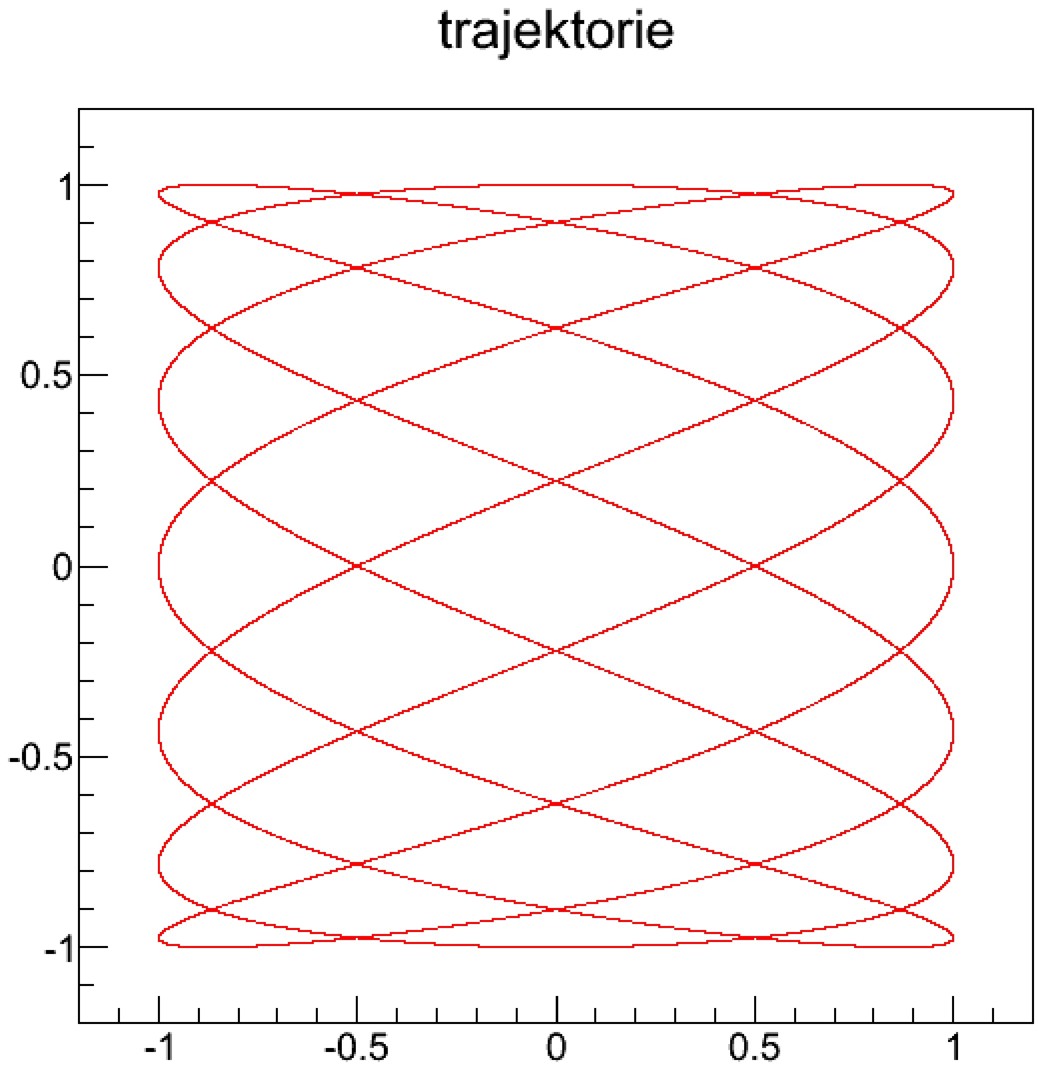
$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 7 \quad \omega_2 = 3$$



Skládání kolmých kmitů

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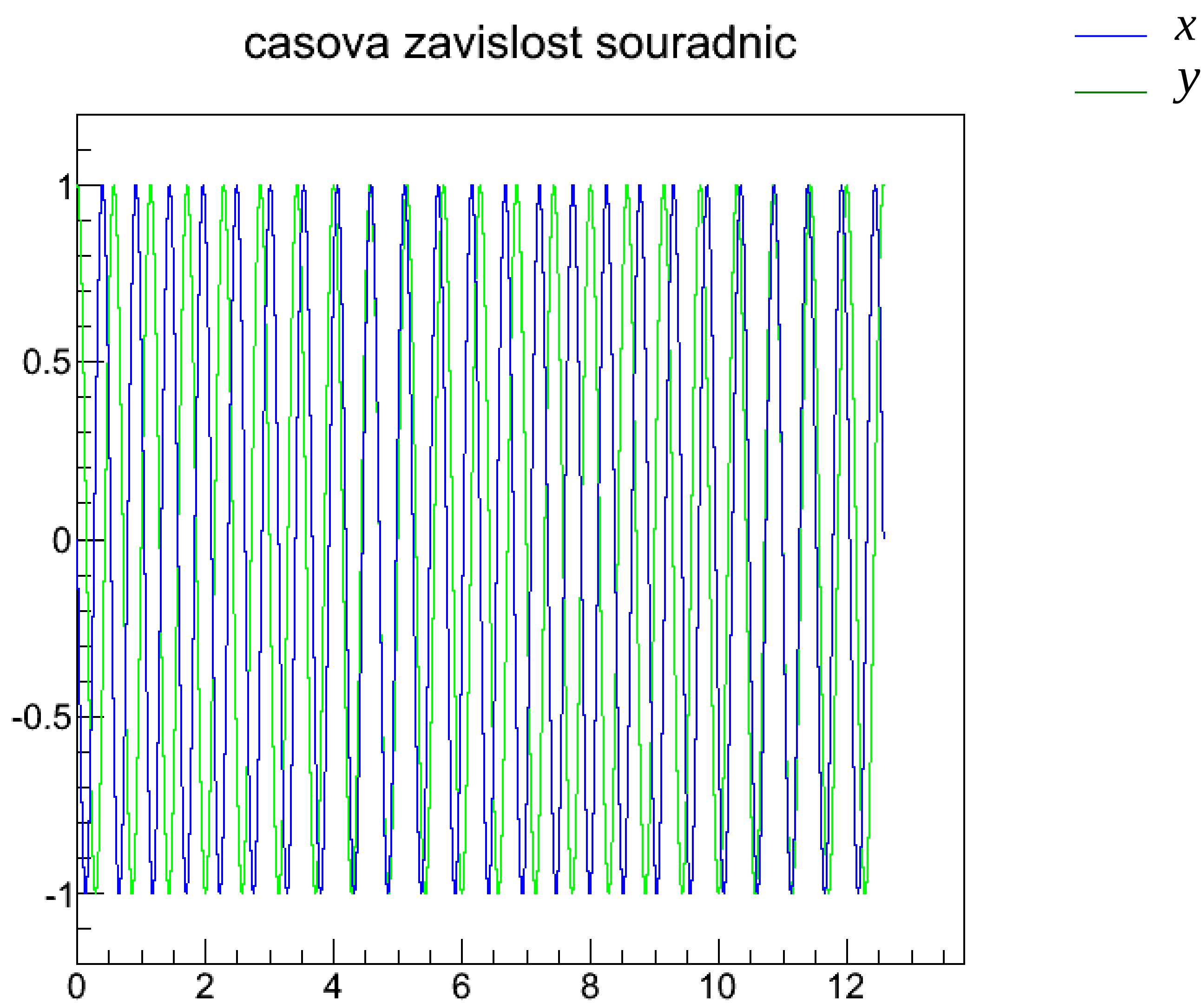
$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 12 \quad \omega_2 = 11$$



Skládání kolmých kmitů

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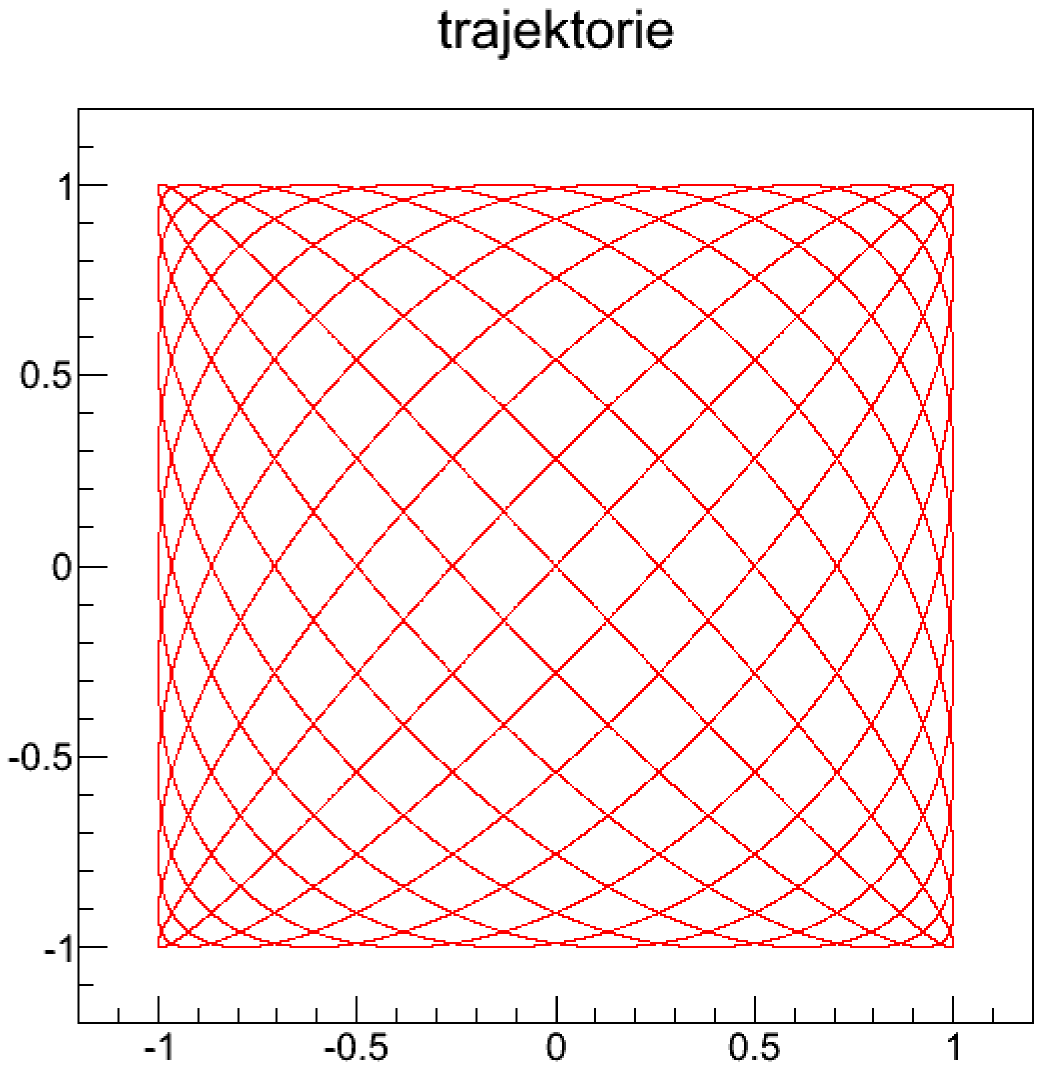
$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = \frac{\pi}{2}$$

$$\omega_1 = 12 \quad \omega_2 = 11$$



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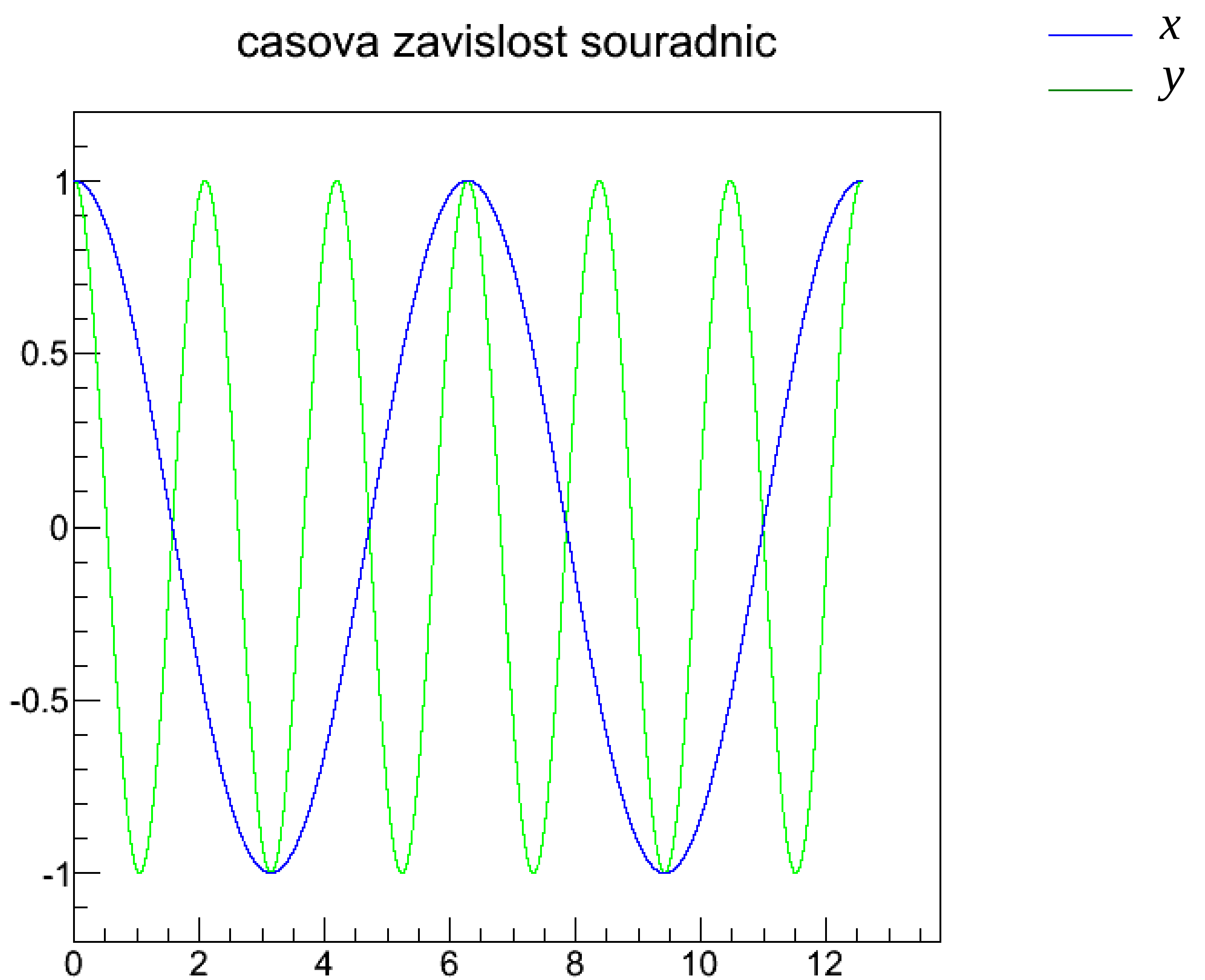
$$y = \hat{A}e^{i\omega_2 t} = A_{0y}e^{i\varphi_2}e^{i\omega_2 t}$$

$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = 0$$

$$\omega_1 = 1 \quad \omega_2 = 3$$



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$$A_{0_x} = 1 \quad A_{0_y} = 1$$

$$\varphi_1 = 0 \quad \varphi_2 = 0$$

$$\omega_1 = 1 \quad \omega_2 = 3$$