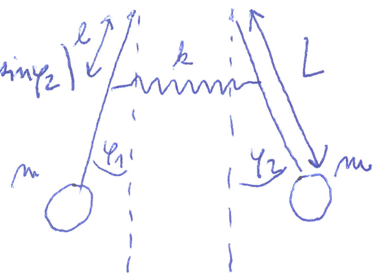
(1) $J\varepsilon = J\ddot{\varphi} = M$

$k\Delta l = k(l\sin\varphi_1 + l\sin\varphi_2)$



$$mL^2\ddot{\varphi}_1 = -mgL\sin\varphi_1 - kl^2(\sin\varphi_1 + \sin\varphi_2)$$
$$mL^2\ddot{\varphi}_2 = -mgL\sin\varphi_2 - kl^2(\sin\varphi_1 + \sin\varphi_2)$$

$\sin\varphi = \varphi$
$$\ddot{\varphi}_1 + \frac{g}{L}\varphi_1 + \frac{kl^2}{mL^2}(\varphi_1 + \varphi_2) = 0$$
$$\ddot{\varphi}_2 + \frac{g}{L}\varphi_2 + \frac{kl^2}{mL^2}(\varphi_1 + \varphi_2) = 0$$

$$\begin{pmatrix} \frac{d^2}{dt^2} + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) & \frac{kl^2}{mL^2} \\ \frac{kl^2}{mL^2} & \frac{d^2}{dt^2} + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

ansatz $\quad \varphi_1 = \Phi_A e^{i\omega t} \qquad \ddot{\varphi}_1 = -\omega^2 \Phi_A e^{i\omega t}$

$\qquad \varphi_2 = \Phi_B e^{i\omega t} \qquad \ddot{\varphi}_2 = -\omega^2 \Phi_B e^{i\omega t}$

$$\begin{pmatrix} -\omega^2 + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) & \frac{kl^2}{mL^2} \\ \frac{kl^2}{mL^2} & -\omega^2 + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) \end{pmatrix} \begin{pmatrix} \Phi_A \\ \Phi_B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\det\begin{pmatrix} -\omega^2 + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) & \frac{kl^2}{mL^2} \\ \frac{kl^2}{mL^2} & -\omega^2 + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right) \end{pmatrix} = 0 \iff \left[-\omega^2 + \left(\frac{g}{L} + \frac{kl^2}{mL^2}\right)\right]^2 - \left(\frac{kl^2}{mL^2}\right)^2 = 0$$

$$\iff -\omega^2 + \frac{g}{L} + \frac{kl^2}{mL^2} = \pm \frac{kl^2}{mL^2}$$

(+) $\quad \omega_1^2 = \frac{g}{L} \Rightarrow \omega_1 = \sqrt{\frac{g}{L}}$

(−) $\quad \omega_2^2 = \frac{g}{L} + \frac{2kl^2}{mL^2} \Rightarrow \omega_2 = \sqrt{\frac{g}{L} + \frac{2kl^2}{mL^2}}$

② $v = \sqrt{\frac{\gamma k T}{m}} = \sqrt{\frac{\gamma R T}{M}}$

$$\frac{v_{He}}{v_{H_2}} = \sqrt{\frac{\gamma_{He} R T}{M_{He}}} \cdot \sqrt{\frac{M_{H_2}}{\gamma_{H_2} R T}} = \sqrt{\frac{\gamma_{He} M_{H_2}}{\gamma_{H_2} M_{He}}}$$

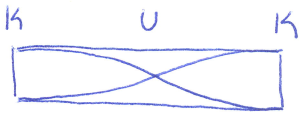
$\gamma = 1 + \frac{2}{f}$ → He je jednoatomová molekula $\Rightarrow f = 3 \Rightarrow \gamma_{He} = \frac{5}{3}$

translace ve 3 směrech

→ H_2 je dvouatomová molekula $\Rightarrow f = 5 \Rightarrow \gamma_{H_2} = \frac{7}{5}$

translace ve 3 směrech
+ rotace kolem 2 os

③



$$\left.\begin{aligned} v &= f\lambda \\ v &= \sqrt{\frac{\gamma k T}{m}} \\ f_2 &= 2 f_1 \end{aligned}\right\}$$

$$\lambda_1 = \lambda_2$$

$$\frac{v_1}{f_1} = \frac{v_2}{f_2}$$

$$\sqrt{\frac{\gamma k T_1}{m}} \frac{1}{f_1} = \sqrt{\frac{\gamma k T_2}{m}} \frac{1}{2 f_1} \quad /^2$$

$$T_1 = \frac{T_2}{4} \Rightarrow \underline{\underline{T_2 = 4 T_1}}$$

(4) a) hvězda se přibližuje $f = f_0 \frac{c}{c - v_s} \Rightarrow v_s = \left(1 - \frac{f_0}{f}\right) c = \left(\frac{f - f_0}{f}\right) c =$

$$= \left(\frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{\lambda}}\right) c = \frac{\lambda_0 - \lambda}{\lambda_0} c$$

hvězda se přibližuje – nastává modrý posuv (posuv k menším vlnovým délkám) – dosazujeme λ_1

$$\underline{\underline{v_s}} = \frac{\lambda_0 - \lambda_1}{\lambda_0} c \doteq \underline{\underline{0{,}38\,c}}$$

b) hvězda se oddaluje $f = f_0 \frac{c}{c + v_s} \Rightarrow v_s = \left(\frac{f_0}{f} - 1\right) c = \left(\frac{f_0 - f}{f}\right) c =$

$$= \left(\frac{\frac{1}{\lambda_0} - \frac{1}{\lambda}}{\frac{1}{\lambda}}\right) c = \frac{\lambda - \lambda_0}{\lambda_0} c$$

hvězda se oddaluje – nastává červený posuv (posuv k větším vlnovým délkám) – dosazujeme λ_2

$$\underline{\underline{v_s}} = \frac{\lambda_2 - \lambda_0}{\lambda_0} c \doteq \underline{\underline{0{,}25\,c}}$$

(5) a) Když se vlak přibližoval, zněli výš. Když se vzdaloval, tak níž.

b) přibližování: $f = f_0 \frac{v}{v - v_s} \Rightarrow v_s = \left(1 - \frac{f_0}{f}\right) v = \left(1 - \frac{1}{\sqrt[12]{2}}\right) v$

$$\underline{\underline{v_s = 69{,}3\ \mathrm{km \cdot h^{-1}}}}$$

vzdalování: $f = f_0 \frac{v}{v + v_s} \Rightarrow v_s = \left(\frac{f_0}{f} - 1\right) v = \left(\sqrt[12]{2} - 1\right) v$

$$\underline{\underline{v_s = 73{,}4\ \mathrm{km \cdot h^{-1}}}}$$

c) přibližování: $f_{z,p} = |f - f_0| = \left| f_0 \left(1 - \frac{v}{v - v_s}\right) \right|$

$$\underline{\underline{f_{z,p} = 7{,}2\ \mathrm{Hz}}}$$

vzdalování: $f_{z,v} = |f - f_0| = \left| f_0 \left(1 - \frac{v}{v + v_s}\right) \right|$

$$\underline{\underline{f_{z,v} = 7{,}0\ \mathrm{Hz}}}$$

⑥

pohybová rovnice pro vzduch v hrdle:

$$m\frac{d^2x}{dt^2} = F = S\Delta p$$

změna tlaku po vychýlení o vzdálenost x

$$pV^\gamma = \text{konst.} \Rightarrow V^\gamma dp + p\gamma V^{\gamma-1}dV = 0$$

$$dp = -\gamma p\frac{dV}{V} = -\gamma p\frac{S\,dx}{V}$$

$$\frac{dp}{p} = -\gamma\frac{S\,dx}{V} \quad /\int$$

$$\ln\frac{p'}{p} = -\gamma\frac{Sx}{V} \Rightarrow p' = pe^{-\gamma\frac{Sx}{V}}$$

$$\Delta p = p' - p = pe^{-\gamma\frac{Sx}{V}} - p = p\Big(\underbrace{e^{-\gamma\frac{Sx}{V}}}_{=1-\gamma\frac{Sx}{V}+\dots} - 1\Big) = -\gamma\frac{Sx}{V}p$$

$$m = \rho S l$$

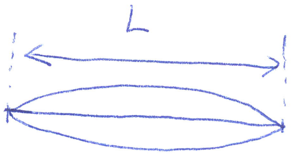
$$m\frac{d^2x}{dt^2} = -\gamma p\frac{S^2}{V}x \Rightarrow \rho S l\frac{d^2x}{dt^2} = -\gamma p\frac{S^2}{V}x$$

$$\frac{d^2x}{dt^2} + \gamma\frac{pS}{\rho l V}x = 0$$

$$\omega = \sqrt{\frac{\gamma p S}{\rho l V}} = v\sqrt{\frac{S}{lV}}$$

v případě lahve od piva $S = \pi r^2$, takže $f = \frac{\omega}{2\pi} = \frac{v}{2\pi}\sqrt{\frac{\pi r^2}{l_1 V}}$

⑦



$\lambda = 2L$

$v = \frac{F_t}{\sigma}$

$v = \lambda f$

$$v^2 = \lambda^2 f^2$$

$$\sigma = \frac{m}{L} = \frac{\rho \cdot V}{L} = \frac{\rho \cdot S \cdot L}{L} = \rho S = \rho\pi\frac{d^2}{4}$$

$$\frac{F_t}{\sigma} = 4L^2f^2 = \frac{F_t}{\rho\pi\frac{d^2}{4}}$$

$$f = \frac{1}{Ld}\sqrt{\frac{F_t}{\pi\rho}}$$

~~7~~8 a) $a_0 = 0 \qquad T = 1$

$$a_n = 2\int_0^{\frac{1}{2}} 1 \cdot \cos(2\pi n x)\,dx - 2\int_{\frac{1}{2}}^{1} 1 \cdot \cos(2\pi n x)\,dx =$$

$$= \frac{1}{2\pi n}\left[\sin(2\pi n x)\right]_0^{\frac{1}{2}} - \frac{1}{2\pi n}\left[\sin(2\pi n x)\right]_{\frac{1}{2}}^{1} = 0$$

$$b_n = 2\cdot\int_0^{\frac{1}{2}} \sin(2\pi n x)\,dx - 2\int_{\frac{1}{2}}^{1} \sin(2\pi n x)\,dx =$$

$$= -\frac{1}{\pi n}\left[\cos(2\pi n x)\right]_0^{\frac{1}{2}} + \frac{1}{\pi n}\left[\cos(2\pi n x)\right]_{\frac{1}{2}}^{1} =$$

$$= -\frac{1}{\pi n}\left[\cos(\pi n) - 1\right] + \frac{1}{\pi n}\left[\cos(2\pi n) - \cos(\pi n)\right] = \begin{cases} 0 & \text{pro } n \text{ sudá} \\ \frac{4}{\pi n} & \text{pro } n \text{ lichá} \end{cases}$$

$$f(x) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[2\pi(2n-1)x]$$

b) $$a_n = 2\int_0^{\frac{1}{4}} 1 \cdot \cos(2\pi n x)\,dx + 2\int_{\frac{1}{4}}^{1} 1 \cos(2\pi n x)\,dx = \frac{2}{2\pi n}\left[\sin(2\pi n x)\right]_0^{\frac{1}{4}} + \frac{2}{2\pi n}\left[\sin(2\pi n x)\right]_{\frac{3}{4}}^{1} =$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{\pi}{2} n\right) - 0\right] + \frac{1}{\pi n}\left[\sin(2\pi n) - \sin\left(\frac{3}{2}\pi n\right)\right]$$

n	1	2	3	4	5	6	7	8
$\sin(\frac{\pi}{2} n)$	1	0	-1	0	1	0	-1	0
$\sin(\frac{3}{2}\pi n)$	-1	0	1	0	-1	0	1	0
a_n	$\frac{2}{\pi \cdot 1}$	0	$-\frac{2}{\pi \cdot 3}$	0	$\frac{2}{\pi \cdot 5}$	0	$-\frac{2}{\pi \cdot 7}$	0

$$\Rightarrow a_n = \frac{2}{\pi} \cdot \frac{(-1)^{n+1}}{2n-1}$$

$$b_n = 2\int_0^{\frac{1}{4}} 1 \cdot \sin(2\pi n x)\,dx + 2\int_{\frac{3}{4}}^{1} 1 \cdot \cos(2\pi n x)\,dx = \frac{2}{2\pi n}\left[\cos(2\pi n x)\right]_0^{\frac{1}{4}} - \frac{2}{2\pi n}\left[\cos(2\pi n x)\right]_{\frac{3}{4}}^{1} =$$

$$= -\frac{1}{\pi n}\left[\cos\left(\frac{\pi}{2} n\right) - 1\right] - \frac{1}{\pi n}\left[1 - \cos\left(\frac{3}{2}\pi\right)\right] = 0$$

$a_0 = \frac{1}{2}$

$$f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos[2\pi(2n-1)x]$$

18 c) $a_n = \int_{-1}^{1} x \cos(\pi n x)\,dx = 0$

(integrujeme součin liché a sudé funkce přes interval symetrický vůči počátku souřadnic)

$$b_n = \int_{-1}^{1} x \sin(\pi n x)\,dx = \left[-x\,\frac{\cos(\pi n x)}{\pi n}\right]_{-1}^{1} + \int_{-1}^{1} \frac{\cos(\pi n x)}{\pi n}\,dx =$$

per partes: $f = x \qquad g = -\frac{\cos(\pi n x)}{\pi n}$

$f' = 1 \qquad g' = \sin(\pi n x)$

$(fg)' = f'g + fg' \quad / \int$

$fg = \int f'g + \int fg'$

$\int fg' = fg - \int f'g$

$$= -2\,\frac{\cos(\pi n)}{\pi n} + \frac{1}{\pi n}\left[\sin(\pi n x)\right]_{-1}^{1} = -2\,\frac{\cos(\pi n)}{\pi n} + \underbrace{\frac{2\sin \pi n}{\pi n}}_{=0} =$$

n	1	2	3	4
$\cos(\pi n)$	-1	1	-1	1

$\Rightarrow -\cos(\pi n) = (-1)^{n+1}$

$$= \frac{2}{\pi n}(-1)^{n+1}$$

$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(\pi n x)$$