Příklad 12.1.

adiabatický děj: $pV^{\gamma} = \text{konst.}$

ideální plyn: $pV = nRT$

$\Rightarrow$ a) $V = \dfrac{nRT}{p} \Rightarrow p\left(\dfrac{nRT}{p}\right)^{\gamma} = \text{konst.}$

$$p^{1-\gamma}\, T^{\gamma} = \text{konst.}$$

$$\underline{T\, p^{\frac{1-\gamma}{\gamma}} = \text{konst.}}$$

$\Rightarrow$ b) $p = \dfrac{nRT}{V} \Rightarrow \dfrac{nRT}{V} V^{\gamma} = \text{konst.}$

$$\underline{T\, V^{\gamma-1} = \text{konst.}}$$

Příklad 12. 2.

adiabatický děj $\qquad T\,p^{\frac{1-\gamma}{\gamma}} = \text{konst.}$

$$T_0\,p_0^{\frac{1-\gamma}{\gamma}} = T_1\,p_1^{\frac{1-\gamma}{\gamma}}$$

$$T_1 = T_0 \cdot \left(\frac{p_0}{p_1}\right)^{\frac{1-\gamma}{\gamma}}$$

$T_1 \doteq 419\ \text{K}$ ... $t_1 \doteq 146\,^\circ\text{C}$

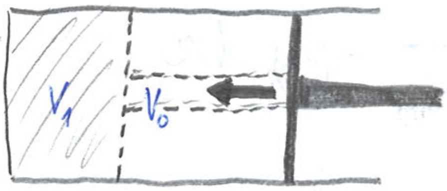
$T_0 = 293\ \text{K}$

$p_0 = 1\ \text{atm.}$

$p_1 = 3{,}5\ \text{atm.}$

$\gamma = \frac{7}{5}$

Příklad 12.3.



$$W = -\int_{1.}^{2.} p\,dV = -\int_{V_0}^{V_1} \frac{nRT}{V}\,dV = -nRT\int_{V_0}^{V_1} \frac{1}{V}\,dV$$

ideální plyn

$pV = nRT$

$p = \frac{nRT}{V}$

$T = $ konst.

$$= -nRT\left[\ln V\right]_{V_0}^{V_1} = -p_0V_0\left(\ln V_1 - \ln V_0\right)$$

izoterm. děj: $p_0V_0 = p_1V_1 = nRT$

$$\underline{W = p_0 V_0 \ln\frac{V_0}{V_1}} \doteq 92{,}8\ \mathrm{kJ}$$

$p_0 = 1$ atm. $= 1{,}01325 \times 10^5$ Pa
$V_0 = 1\,m^3$
$V_1 = 0{,}4\,m^3$

Příklad 12.4.

- atmosférický tlak $\quad dp = -\rho g\, dz \quad$ - pokles tlaku při vystoupání o výšku dz

- adiabatický děj $\quad pV^{\gamma} = \text{konst.} \iff p\rho^{-\gamma} = \text{konst.}$

$$TV^{\gamma-1} = \text{konst.} \iff T\rho^{1-\gamma} = \text{konst.} \quad (*)$$

totální diferenciál

$$dp\,\rho^{-\gamma} - \gamma p \rho^{-\gamma-1} d\rho = 0$$

$$dp = \frac{\gamma p}{\rho} d\rho$$

$$\frac{\gamma p}{\rho} d\rho = -\rho g\, dz$$

$$\gamma \frac{p}{\rho^2} d\rho = -g\, dz$$

$$p\rho^{-\gamma} = \text{konst.} = p_0\rho_0^{-\gamma} \longrightarrow \gamma p_0 \rho_0^{-\gamma} \rho^{+\gamma} \rho^{-2} d\rho = -g\, dz$$

$$p = p_0 \rho_0^{-\gamma} \cdot \rho^{+\gamma}$$

separace proměnných

$$\int_{\rho_0}^{\rho(h)} \gamma p_0 \rho_0^{-\gamma} (\rho(h))^{-2+\gamma} d\rho = -\int_0^h g\, dz$$

$$\gamma p_0 \rho_0^{-\gamma} \left[\frac{\rho^{\gamma-1}}{\gamma-1} \right]_{\rho_0}^{\rho(h)} = -g\, [z]_0^h$$

$$\gamma p_0 \rho_0^{-\gamma} \frac{1}{\gamma-1} \left(\rho^{\gamma-1} - \rho_0^{\gamma-1} \right) = -g h$$

$$\rho^{\gamma-1} = \rho_0^{\gamma-1} - \frac{hg}{p_0}\frac{\gamma}{\gamma-1}\rho_0^{+\gamma}\cdot\frac{\rho_0^{\gamma-1}}{\rho_0^{\gamma-1}}$$

$$\underline{\rho^{\gamma-1} = \rho_0^{\gamma-1}\left(1 - \frac{\rho_0 hg}{p_0}\frac{\gamma}{\gamma-1}\right)}$$

$\downarrow$ adiabatický děj (*)

$$T\rho^{1-\gamma} = \text{konst.}$$

$$T(h)\rho(h)^{1-\gamma} = T_0\rho_0^{1-\gamma}$$

$$\Rightarrow T(h) = T_0\cdot\frac{\rho_0^{1-\gamma}}{\rho(h)^{1-\gamma}} = T_0\frac{\rho(h)^{\gamma-1}}{\rho_0^{\gamma-1}}$$

$$\underline{\underline{T(h) = T_0\cdot\left(1 - \frac{\rho_0 hg}{p_0}\frac{\gamma}{\gamma-1}\right)}}$$

Příklad 12.5.

adiabatický děj $\quad pV^\gamma = \text{konst.}$

$$p_0 V_0^\gamma = p_1 V_1^\gamma$$

a) $$p_0 V_0^\gamma = p_1 \left(\tfrac{1}{2} V_0\right)^\gamma$$

$$p_0 = p_1 \frac{1}{2^\gamma}$$

$$\underline{p_1 = p_0 \cdot 2^\gamma}$$

$$\gamma_A = \frac{5}{3} \Rightarrow p_{1A} = 3{,}17\, p_0$$
$$\gamma_B = \frac{7}{5} \Rightarrow p_{1B} = 2{,}64\, p_0$$

b) $$W = -\int_{1.}^{2.} p\, dV = -p_0 V_0^\gamma \int_{V_0}^{V_1} V^{-\gamma}\, dV$$

$$p = \frac{\text{konst.}}{V^\gamma}$$

$$p = \frac{p_0 V_0^\gamma}{V^\gamma}$$

$$= -p_0 V_0^\gamma \left[\frac{1}{-\gamma+1} V^{-\gamma+1}\right]_{V_0}^{V_1} = -\frac{p_0 V_0^\gamma}{1-\gamma} V_0^{-\gamma+1}\left(\frac{1}{2^{-\gamma+1}} - 1\right)$$

$$\underline{W = \frac{p_0 V_0}{\gamma - 1}\left(2^{\gamma-1} - 1\right)}$$

$$\gamma_A = \frac{5}{3} \Rightarrow W_A = 0{,}88\, p_0 V_0$$

$$\gamma_B = \frac{7}{5} \Rightarrow W_B = 0{,}80\, p_0 V_0$$

Příklad 12.6.

1. termodynamický zákon $\quad Q = \Delta U + W$

$$\underbrace{n C_p (T_1 - T_0)}_{\text{⊛ izobarický děj}} = \underbrace{n C_V (T_1 - T_0)}_{\text{změna vnitřní energie}} + \int_{\text{stav. 0.}}^{\text{stav. 1.}} p \, dV$$

izobarický děj $p = \text{konst.}$

$$n C_p (T_1 - T_0) = n C_V (T_1 - T_0) + p_0 V_1 - p_0 V_0$$

ideální plyn

$$p_0 V_1 = nRT_1 \qquad p_0 V_0 = nRT_0$$

$$n C_p (T_1 - T_0) = n C_V (T_1 - T_0) + nR (T_1 - T_0)$$

$$\Rightarrow \underline{C_p = C_V + R}$$

⊛ $\quad p_0, V_0, T_0 \longrightarrow \underline{\underline{p_0}}, V_1, T_1$

Příklad 12.7.

$T = T_0$:

$$p_0 V_1 = n_1 R T_0 \qquad p_0 V_2 = n_2 R T_0$$

$T = \begin{cases} T_1 \\ T_2 \end{cases}$:

$$p V_1 = n_1' R T_1 \qquad p V_2 = n_2' R T_2$$

VÝMĚNA LÁTKY

(*) $n_1 = \dfrac{p_0 V_1}{R T_0}$ (*) $n_2 = \dfrac{p_0 V_2}{R T_0}$

$$n_1' = \frac{p V_1}{R T_1} \qquad n_2' = \frac{p V_2}{R T_2}$$

STEJNÝ TLAK

=> 1. rovnice:

$$p = \frac{n_1' R T_1}{V_1} = \frac{n_2' R T_2}{V_2}$$

2. rovnice:

$$n_1 + n_2 = n_1' + n_2'$$

$$n_2' = n_1 + n_2 - n_1'$$

$$n_1' \frac{T_1}{V_1} = (n_1 + n_2)\frac{T_2}{V_2} - n_1' \frac{T_2}{V_2}$$

$$n_1' = (n_1 + n_2)\frac{T_2}{V_2} \Big/ \left(\frac{T_1}{V_1} + \frac{T_2}{V_2}\right)$$

$$n_1' = (n_1 + n_2)\frac{T_2 V_1}{T_1 V_2 + T_2 V_1}$$

$$n_2' = (n_1 + n_2)\frac{T_1 V_2}{T_1 V_2 + T_2 V_1}$$

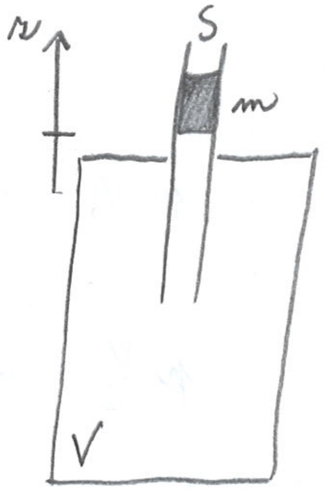
$$\Rightarrow p = \frac{n_1' R T_1}{V_1} = (n_1 + n_2) R \frac{T_1 T_2}{T_1 V_2 + T_2 V_1} \overset{(*)}{=} \frac{p_0}{T_0} T_1 T_2 \frac{V_1 + V_2}{T_1 V_2 + T_2 V_1}$$

$$p \doteq 1{,}108\, p_0 \doteq 1{,}12 \text{ kPa}$$

$T_0 = 300$ K

$T_1 = 373$ K $\quad T_2 = 273$ K

Příklad 12.8.



- rovnovážný tlak $\quad \underline{p_0 = p_a + \frac{mg}{S}}$

- změna tlaku (závaží dolů) $\quad \Delta p \quad \Rightarrow$ tlaková síla $\underline{F = S \cdot \Delta p}$

  - adiabatický děj $\quad pV^\gamma = \text{konst.}$

    totální diferenciál $\quad dp\,V^\gamma + p\,\gamma\,V^{\gamma-1}\,dV = \text{konst.}$

$$\Rightarrow dp = -\frac{\gamma p}{V}\,dV$$

$$\underline{\Delta p = -\frac{\gamma p_0}{V}\,\Delta V}$$

  - změna objemu $\quad \underline{\Delta V = S \cdot z}$

- 2. Newtonův zákon $\quad m\ddot{z} = F$

$$m\ddot{z} = -S \cdot \frac{\gamma p_0}{V} \cdot Sz$$

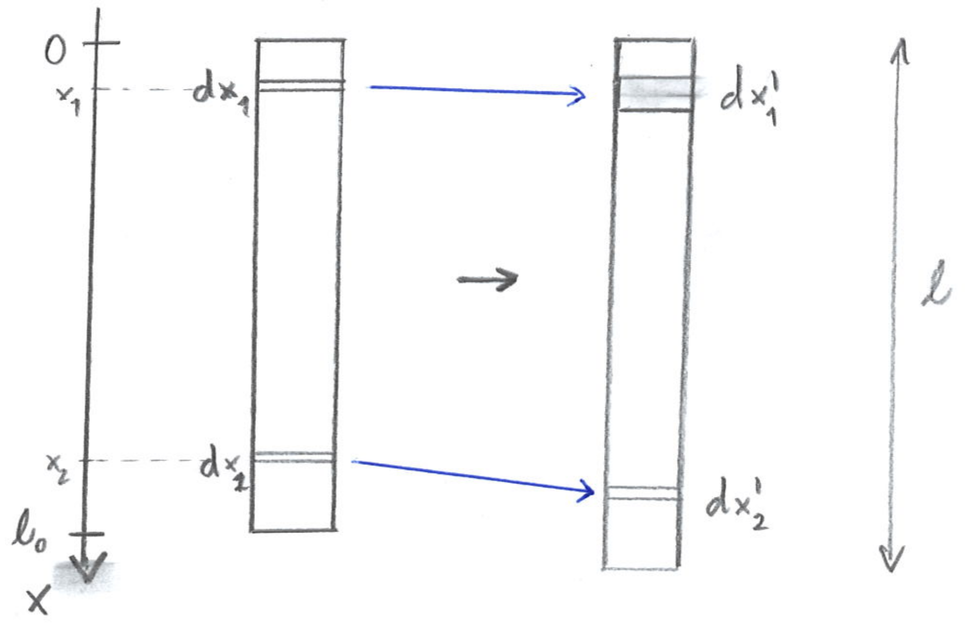
$$\underline{\ddot{z} + \frac{\gamma S^2 p_0}{mV} z = 0} \quad \Leftrightarrow \text{harmonický oscilátor}$$

$$\ddot{z} + \omega^2 z = 0$$

$$\Rightarrow \omega = \sqrt{\frac{\gamma S^2 p_0}{mV}}$$

$$\gamma = \omega^2 \frac{mV}{S^2 p_0} = \underline{\frac{4\pi^2}{T^2} \frac{mV}{S^2 p_0}}$$

Příklad 12.9.



• Hookův zákon $\varepsilon = \dfrac{\sigma}{E} = \dfrac{1}{E} \cdot \dfrac{F}{S}$ → síla části tyče pod elementem dx

$$\varepsilon(x) = \frac{1}{SE} \cdot mg \, \frac{l_0 - x}{l_0}$$

• deformace (v místě x) $\varepsilon(x) = \dfrac{dx' - dx}{dx} \Rightarrow dx' = (1+\varepsilon(x))\,dx$

počáteční délka l_0

$$l = \int_0^{l} dx' = \int_0^{l_0} (1+\varepsilon(x))\,dx$$

konečná délka

$$l = \int_0^{l_0} \left(1 + \frac{mg}{SE}\,\frac{l_0 - x}{l_0}\right) dx$$

$$= l_0 + \frac{mg}{SE}\,\frac{1}{l_0}\left[l_0 x - \frac{x^2}{2}\right]_0^{l_0} = l_0 + \frac{mg}{SE}\,\frac{l_0}{2}$$

$$\Rightarrow \underline{l = l_0 + \frac{1}{2} l_0 \frac{mg}{SE}}$$

$$\underline{\varepsilon = \frac{l - l_0}{l_0} = \frac{1}{2}\,\frac{mg}{SE}}$$