

Anihilace pozitronů v pevných látkách

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Doporučená literatura:

- P.Hautojärvi: Positrons in Solids, Topics in Current Physics, Springer-Verlag (1979)
- A. Dupasquier, A.P. Mills, Jr. (eds.): Positron Spectroscopy of Solids, IOS Press, Amsterdam (1995)
- R. Krause-Rehberg, H.S. Leipner, Positron Annihilation in Semiconductors – Defect Studies, Springer, Berlin (1999)
- P.J. Schultz, K.G. Lynn, Interaction of positron beams with surfaces, thin films, and interfaces, Rev. Mod. Phys. 60, 701 (1988)
- M.J. Puska, R.M. Nieminen, Theory of positrons in solids and solid surfaces, Rev. Mod. Phys. 66, 841 (1994)

Pozitron – teoretická předpověď

Schrödingerova rovnice:
$$\left[\frac{\hat{\mathbf{p}}^2}{2m} + V(\mathbf{x}, t) \right] \psi(\mathbf{x}, t) = i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t}$$

- nerelativistická pohybová rovnice pro elektron



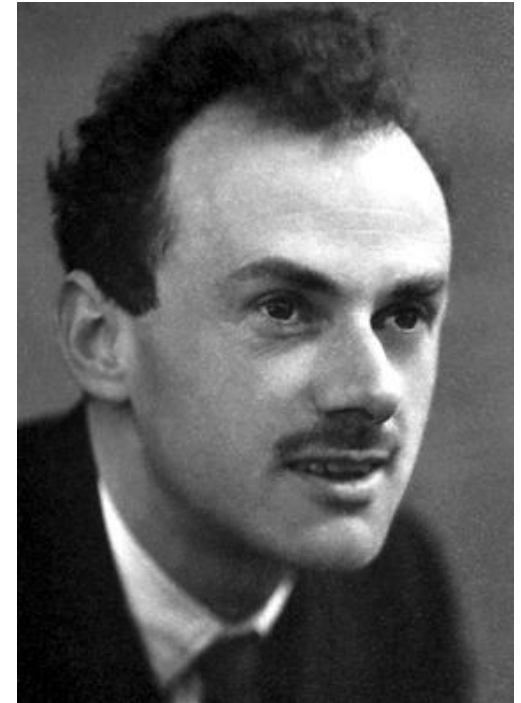
Erwin Schrödinger
1933 Nobelova cena

Pozitron – teoretická předpověď

Diracova rovnice:
$$(\boldsymbol{\alpha} \hat{\mathbf{p}} c + \beta mc^2) \psi(\mathbf{x}, t) = i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t}$$

- relativistická pohybová rovnice pro pozitron
- řešení s kladnou energií: 'normální elektrony'
- řešení se zápornou energií

- kinetická energie částice
$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m} \quad (\text{klasicky})$$



Paul Adrien Maurice Dirac
1933 Nobelova cena

P.A.M. Dirac, Proc. R. Soc. Lond. A 117, 610-624 (1928)

P.A.M. Dirac, Proc. R. Soc. Lond. A 133, 60-72 (1931)

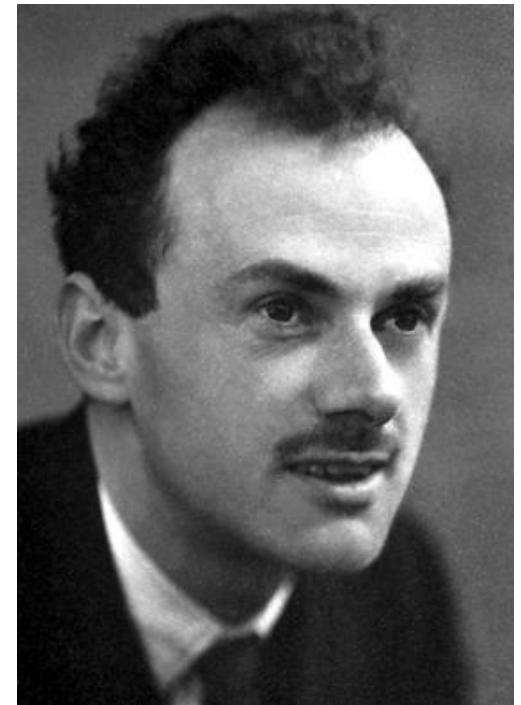
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• relativistická energie
$$E^2 = m^2 c^4 + p^2 c^2$$

$$E = \pm \sqrt{m^2 c^4 + p^2 c^2}$$



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1933 Nobelova cena

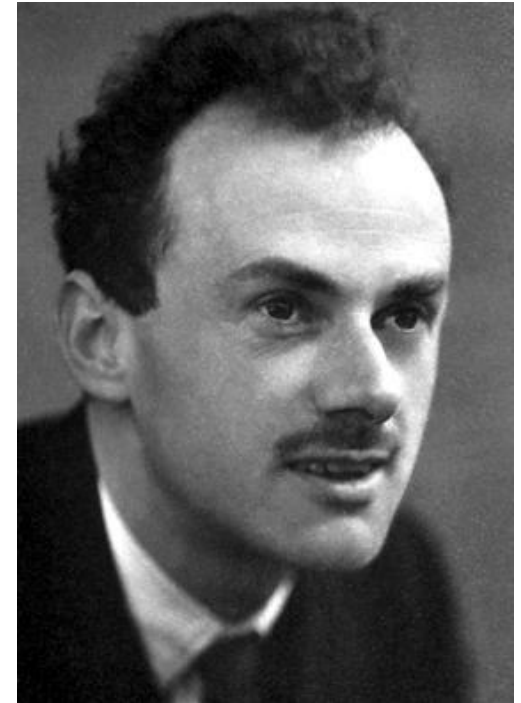
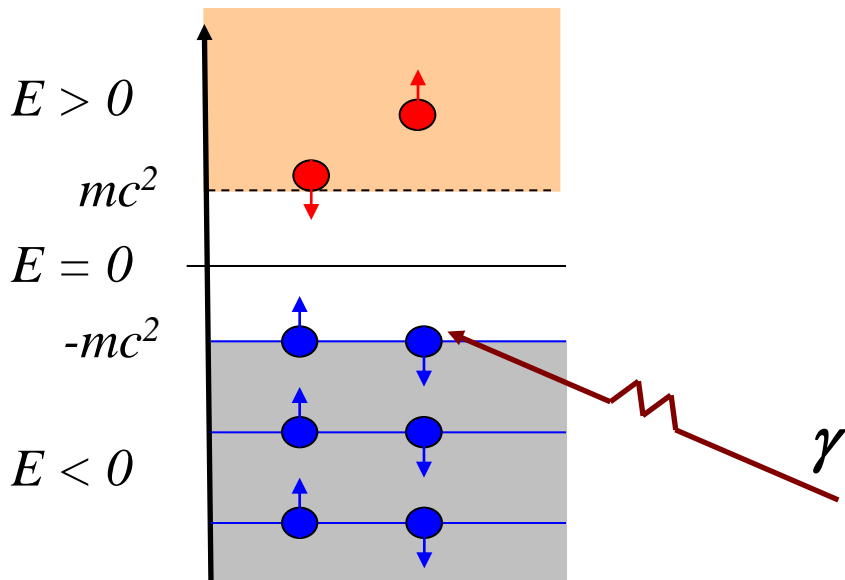
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- relativistická pohybová rovnice pro pozitron
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- vakuum je moře elektronů se zápornou energií



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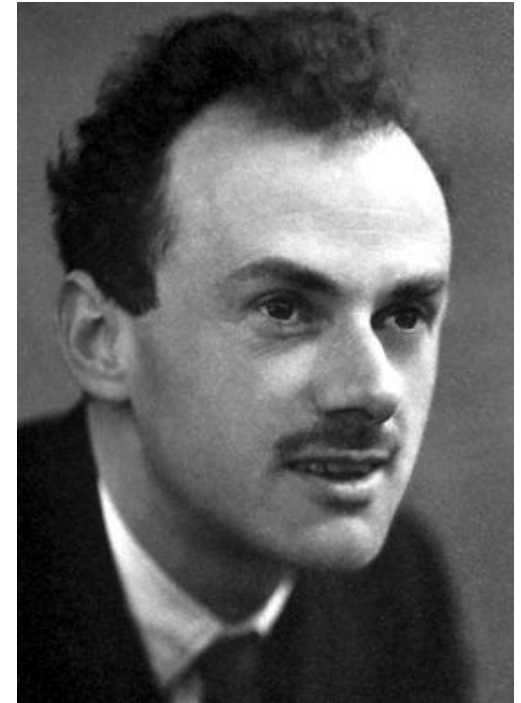
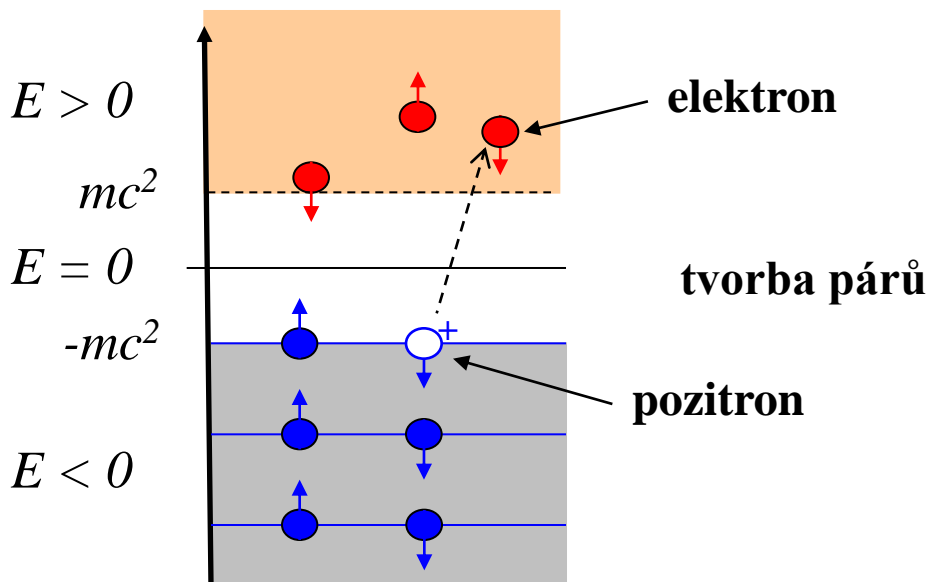
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- pozitron je "díra" ve vakuu



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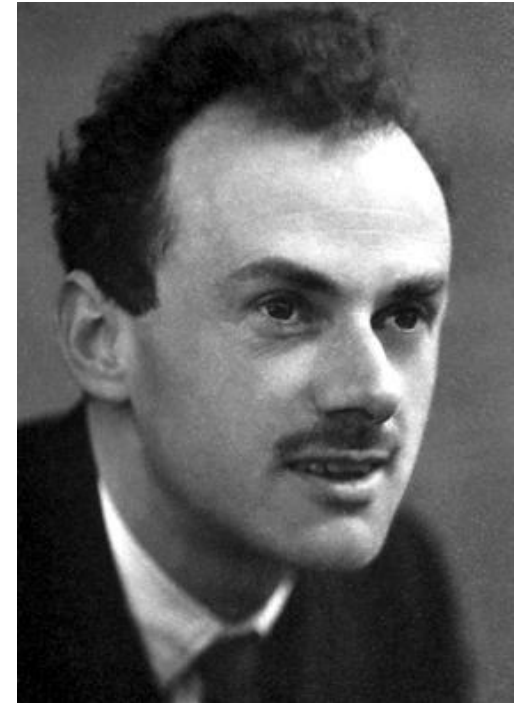
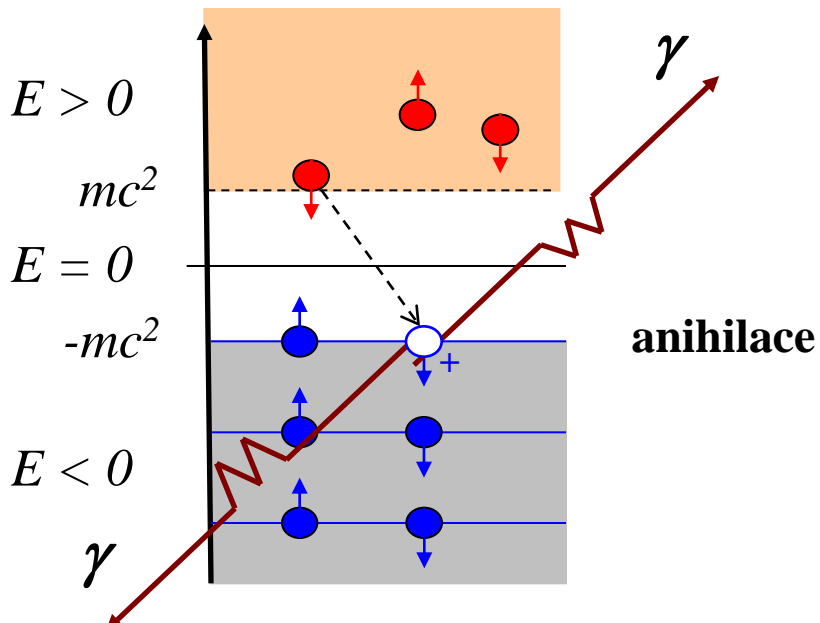
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Objev pozitronu

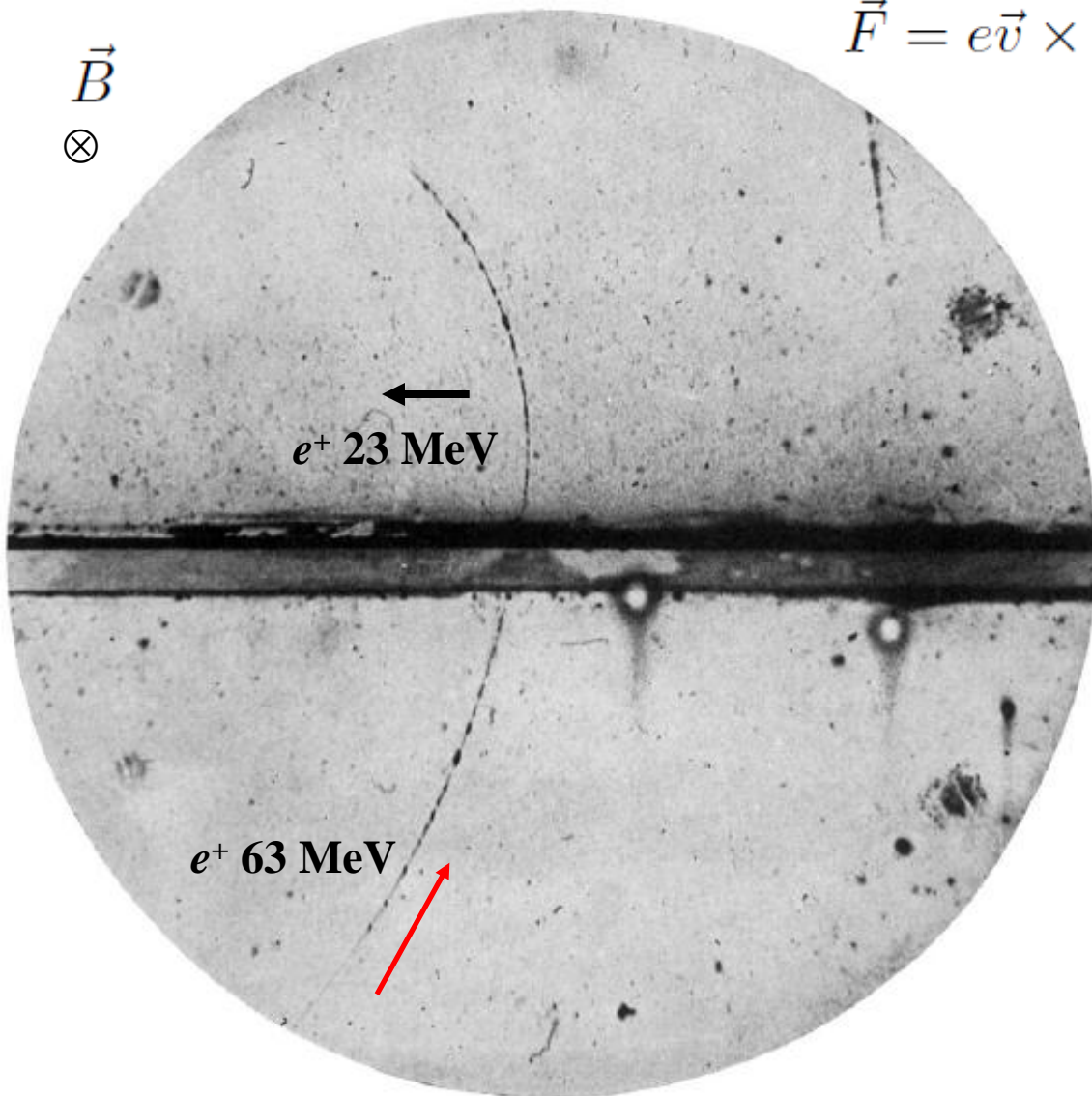
Objev pozitronu 1932

Lorentzova síla

$$\vec{F} = e\vec{v} \times \vec{B}$$

\vec{B}

⊗



6 mm
Pb folie



Carl David Anderson

1936 Nobelova cena

The Positive Electron

CARL D. ANDERSON, *California Institute of Technology, Pasadena, California*

(Received February 28, 1933)

Out of a group of 1300 photographs of cosmic-ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than twice, and is probably exactly equal to, that of the proton. If these particles carry unit positive charge the

curvatures and ionizations produced require the mass to be less than twenty times the electron mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.

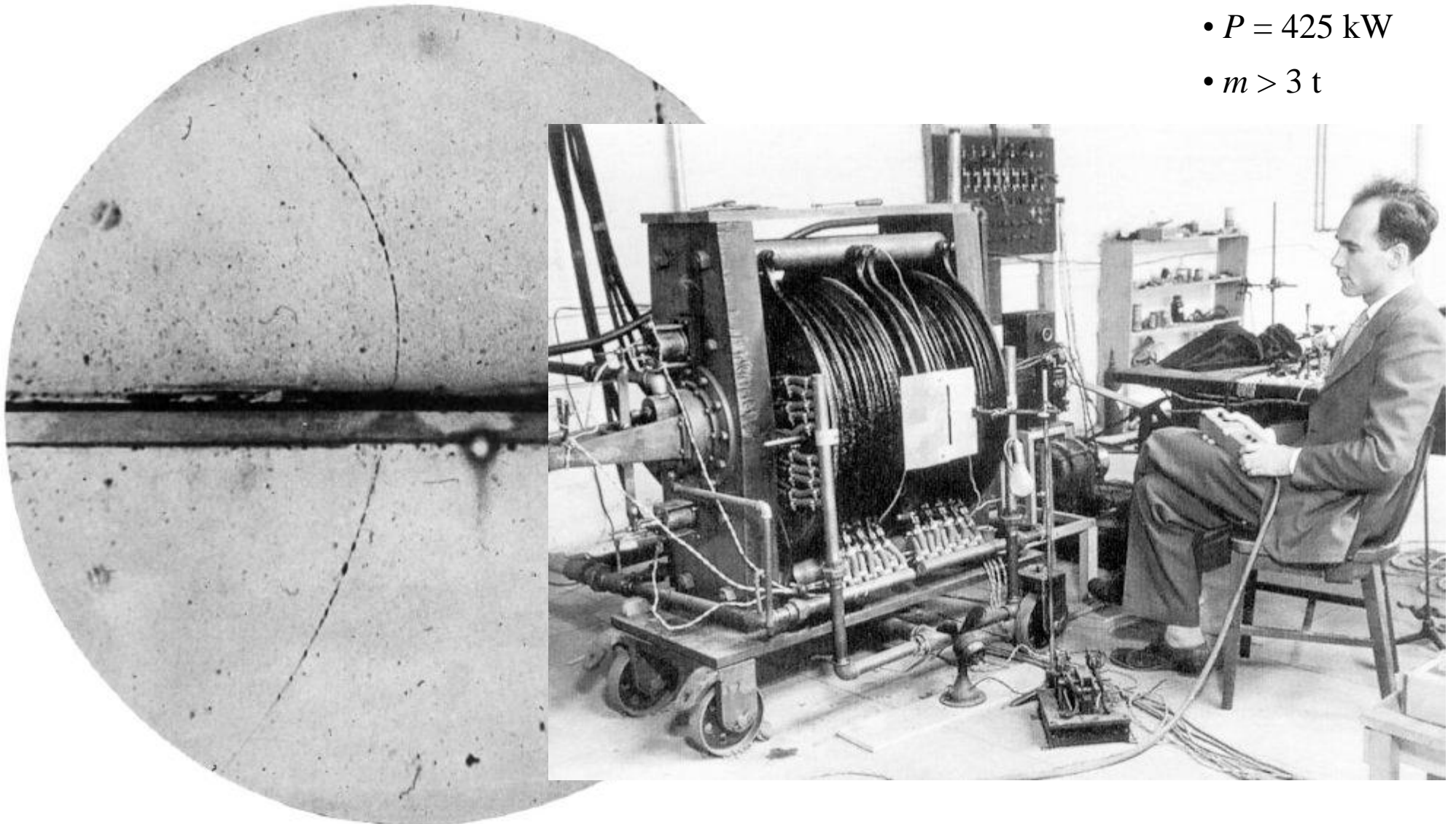
Editor

ON August 2, 1932, during the course of photographing cosmic-ray tracks produced in a vertical Wilson chamber (magnetic field of 15,000 gauss) designed in the summer of 1930 by Professor R. A. Millikan and the writer, the tracks shown in Fig. 1 were obtained, which seemed to be interpretable only on the basis of the existence in this case of a particle carrying a positive charge but having a mass of the same order of magnitude as that normally possessed by a free negative electron. Later study of the

electrons happened to produce two tracks so placed as to give the impression of a single particle shooting through the lead plate. This assumption was dismissed on a probability basis, since a sharp track of this order of curvature under the experimental conditions prevailing occurred in the chamber only once in some 500 exposures, and since there was practically no chance at all that two such tracks should line up in this way. We also discarded as completely untenable the assumption of an electron of 20

Objev pozitronu

- $B = 1.7 \text{ T}$
- $P = 425 \text{ kW}$
- $m > 3 \text{ t}$



Pozitron – historie objevu

1929 - Dmitri Vladimirovich Skobeltsyn (St. Peterburg)

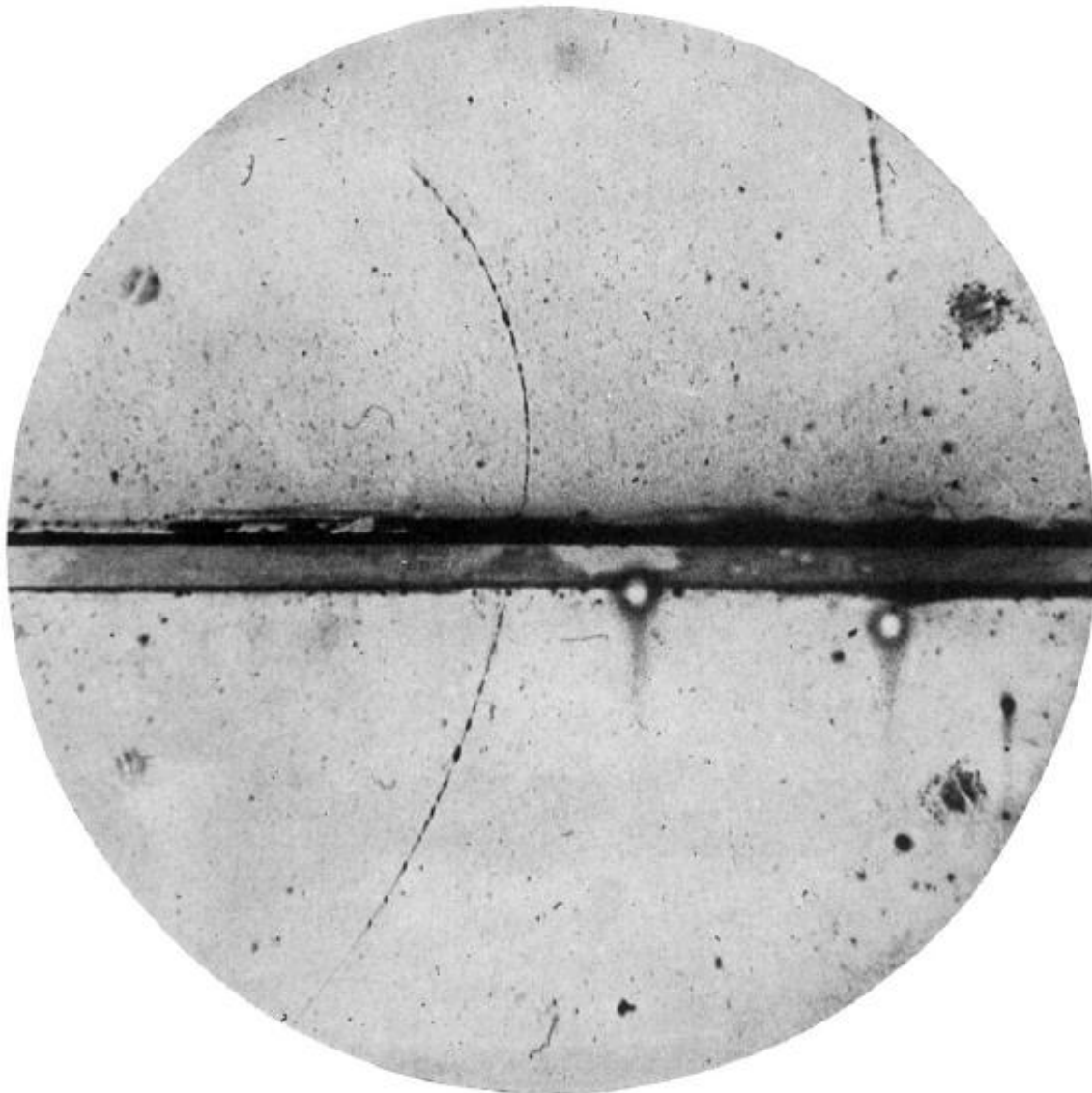
1929 - Chung-Yao-Chao (California Institute of Technology)

2. 8. 1932 – Carl David Anderson (California Institute of Technology)

1936 – Carl David Anderson a jeho student Seth Neddermeyer objevili mion

C.D. Anderson: The Positive Electron, Phys. Rev. 43, 491 (1933)

Pozitron

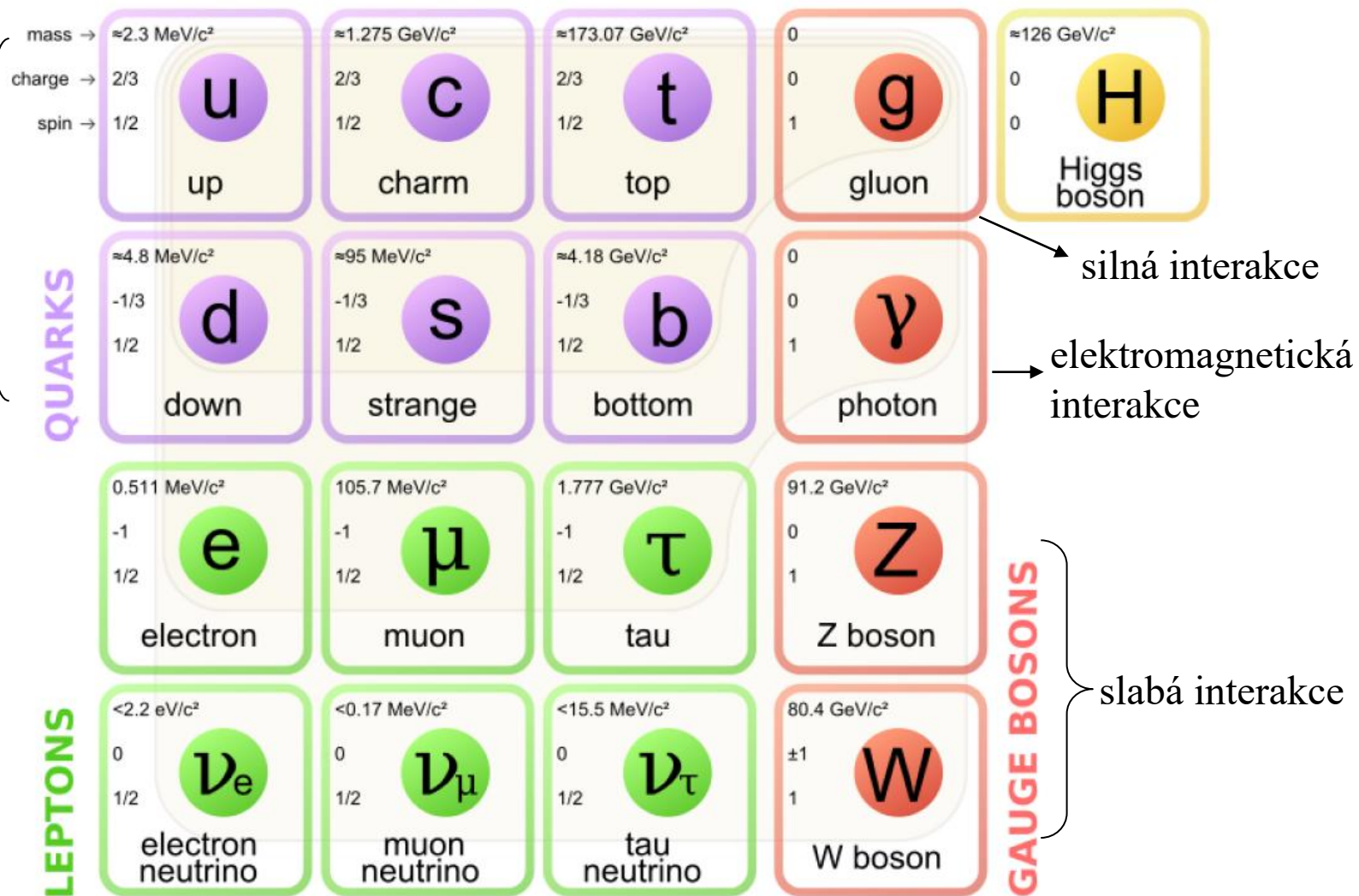


pozitron = antičástice elektronu

- klidová hmotnost: m_e
- náboj: $+e$
- spin: $1/2$

Elementární částice (standardní model)

tvoří hadrony
protony, neutrony,
mesony, baryony



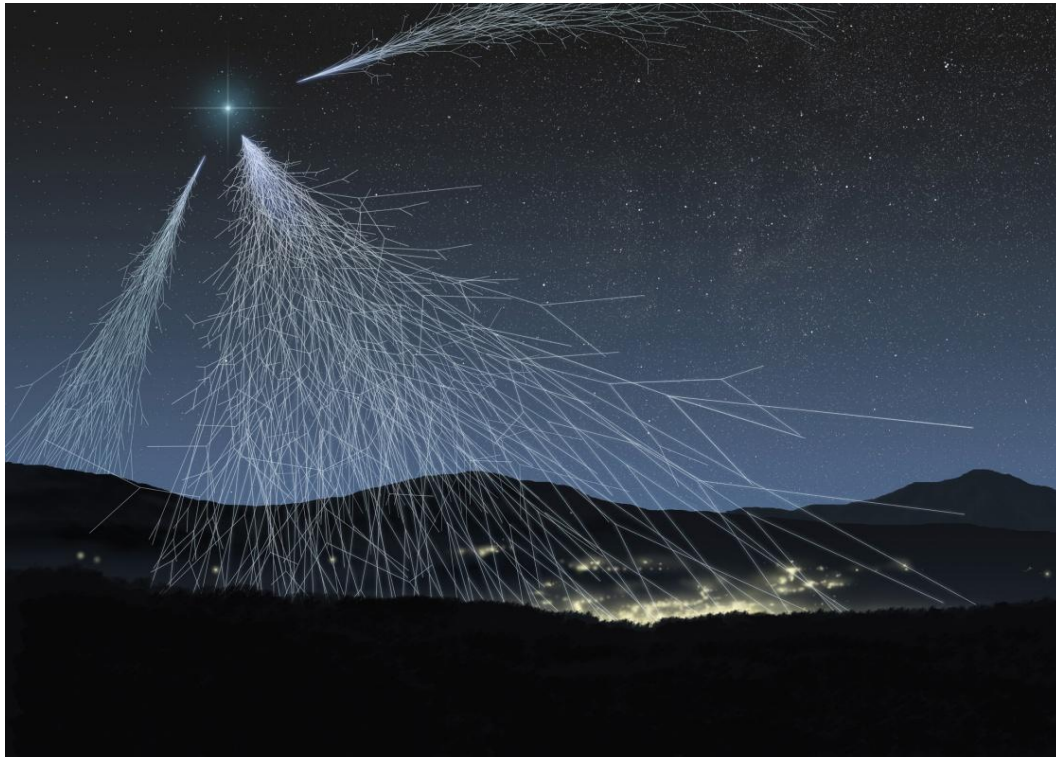
Zdroje pozitronů

kosmické záření

90 % protony

9 % α -částice

1 % těžší jádra & ostatní částice (e^- , e^+ , p^-)



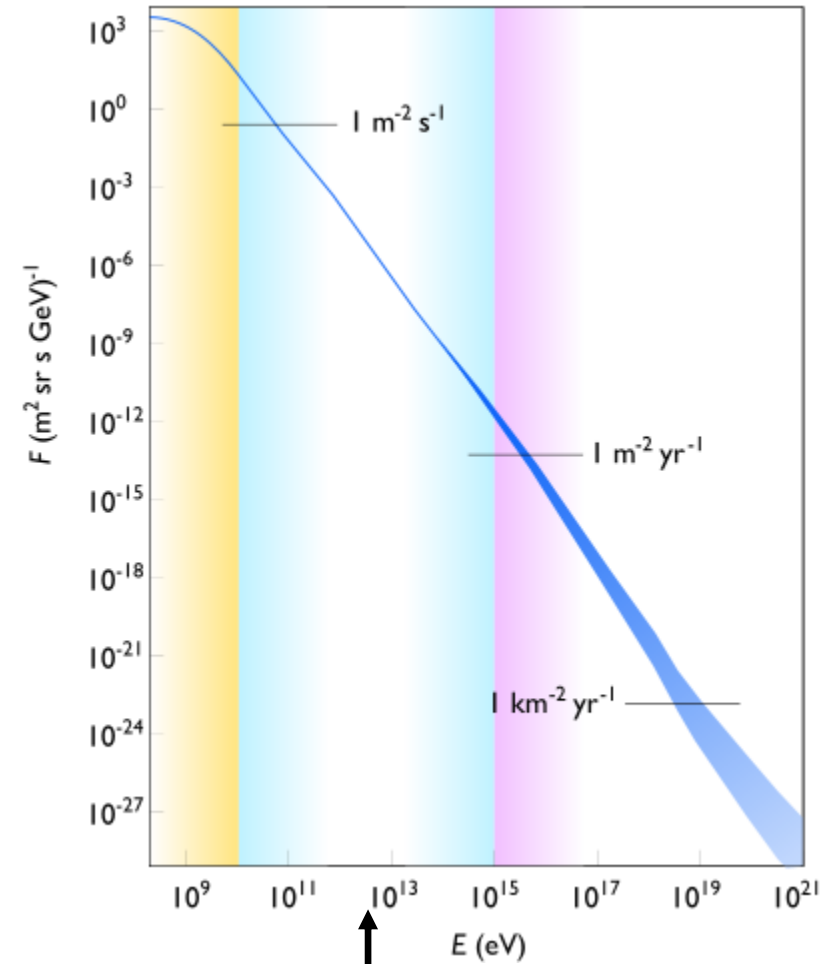
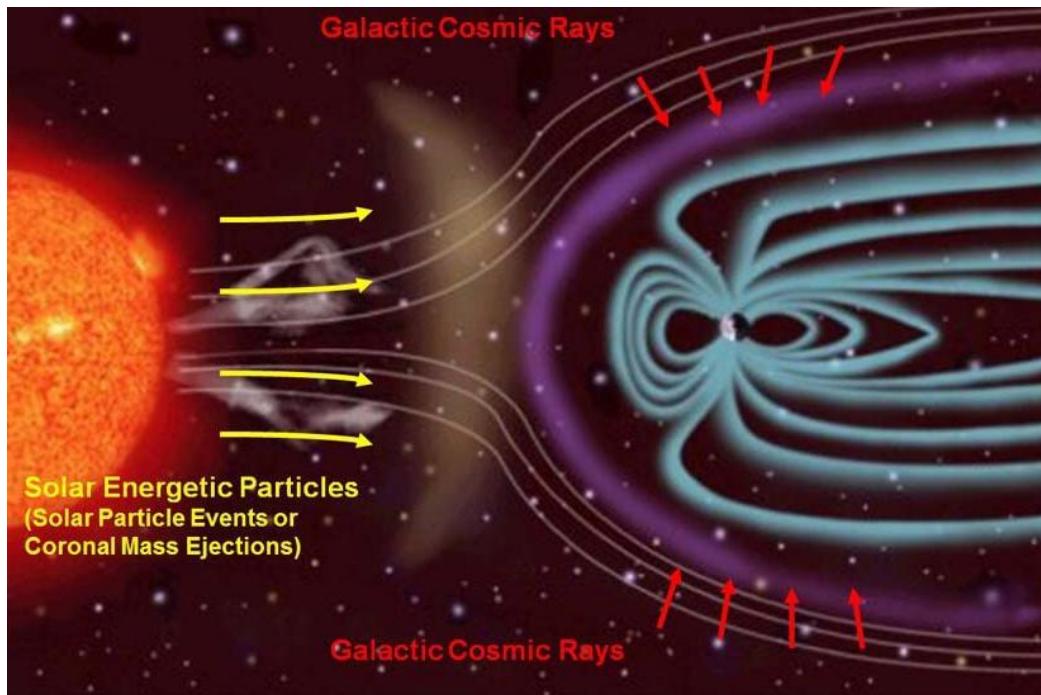
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7 TeV (LHC, CERN)

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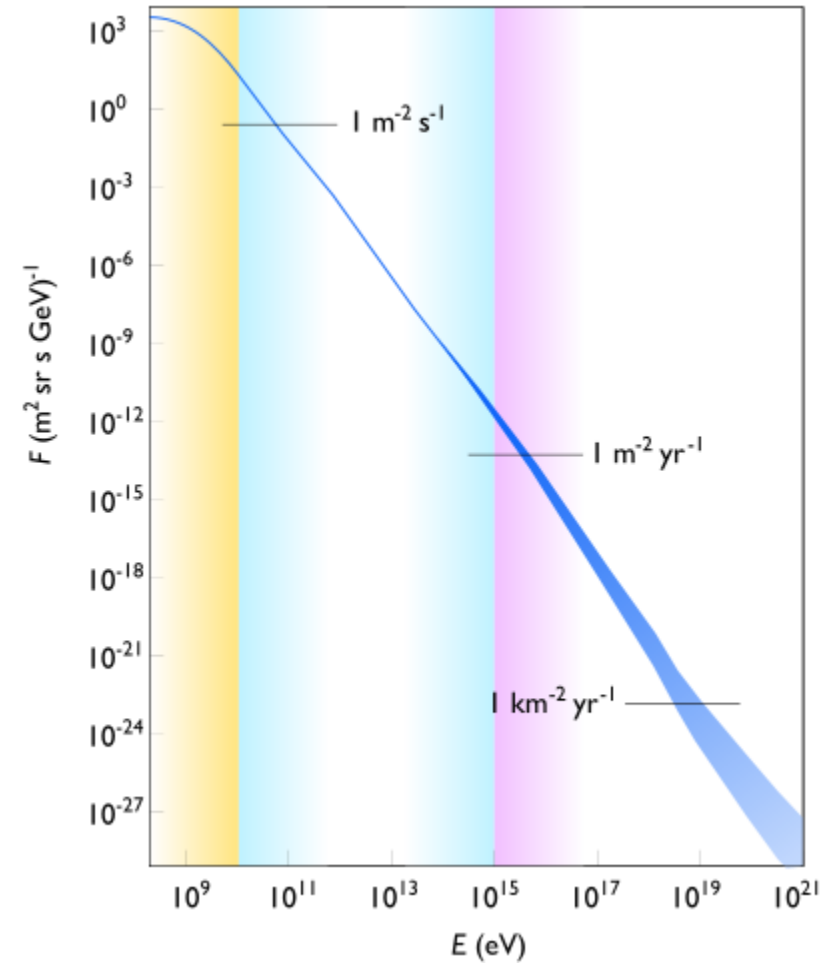
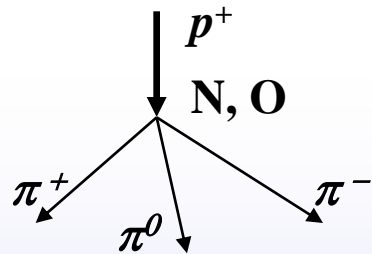
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interakce s
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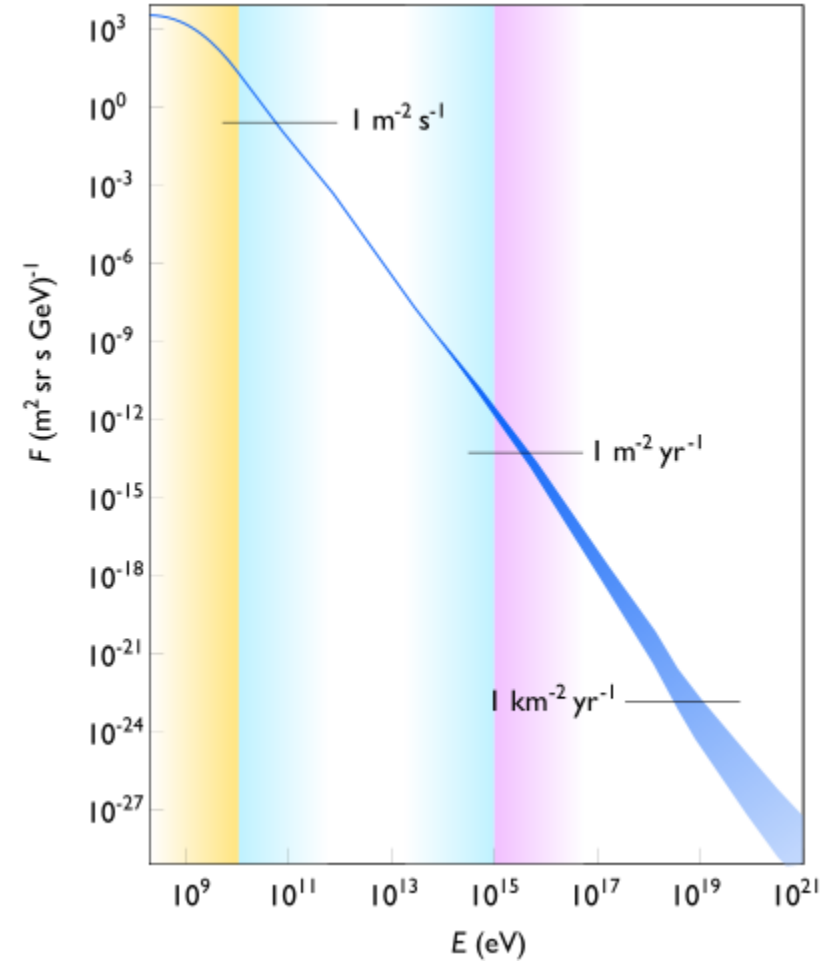
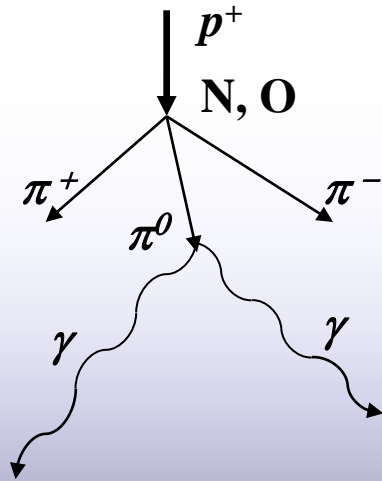
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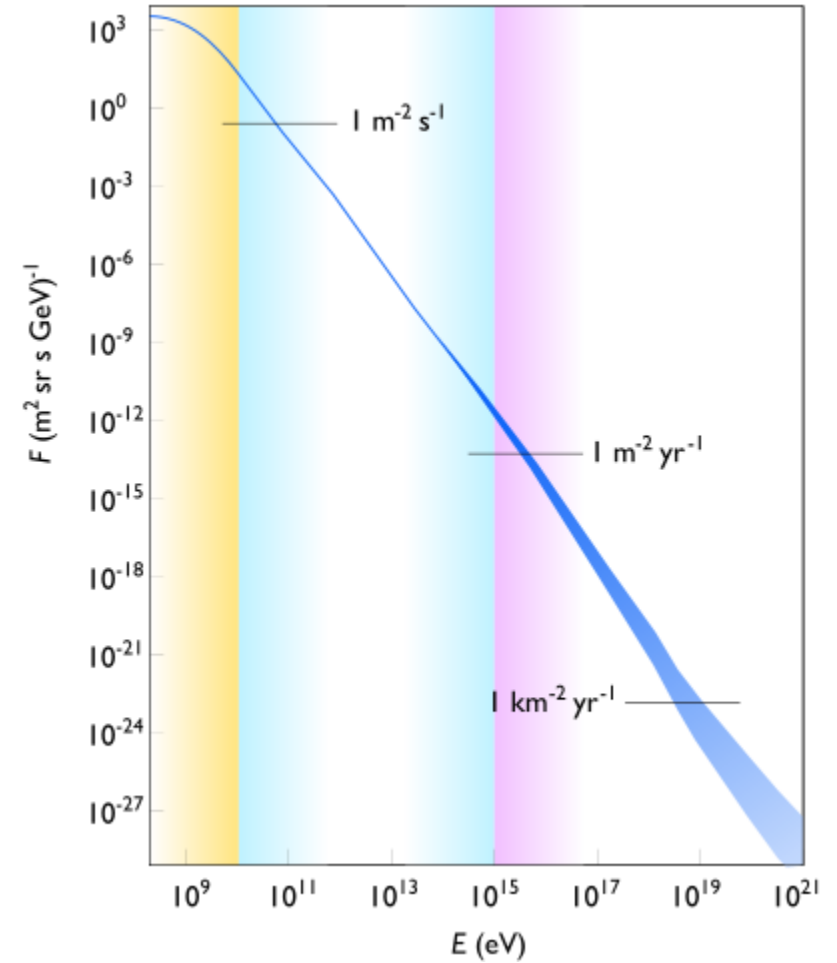
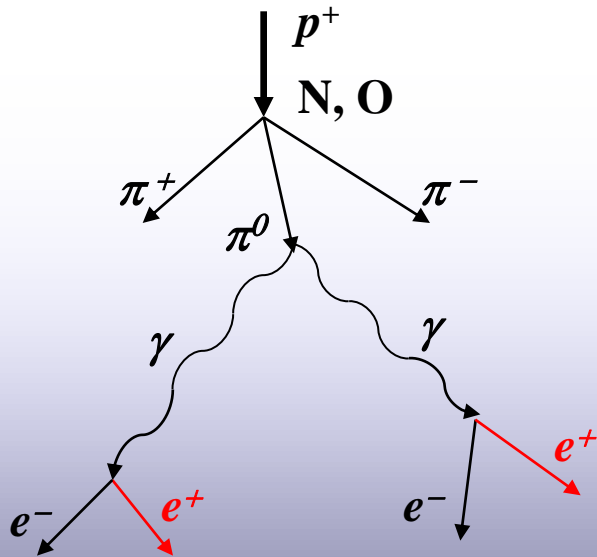
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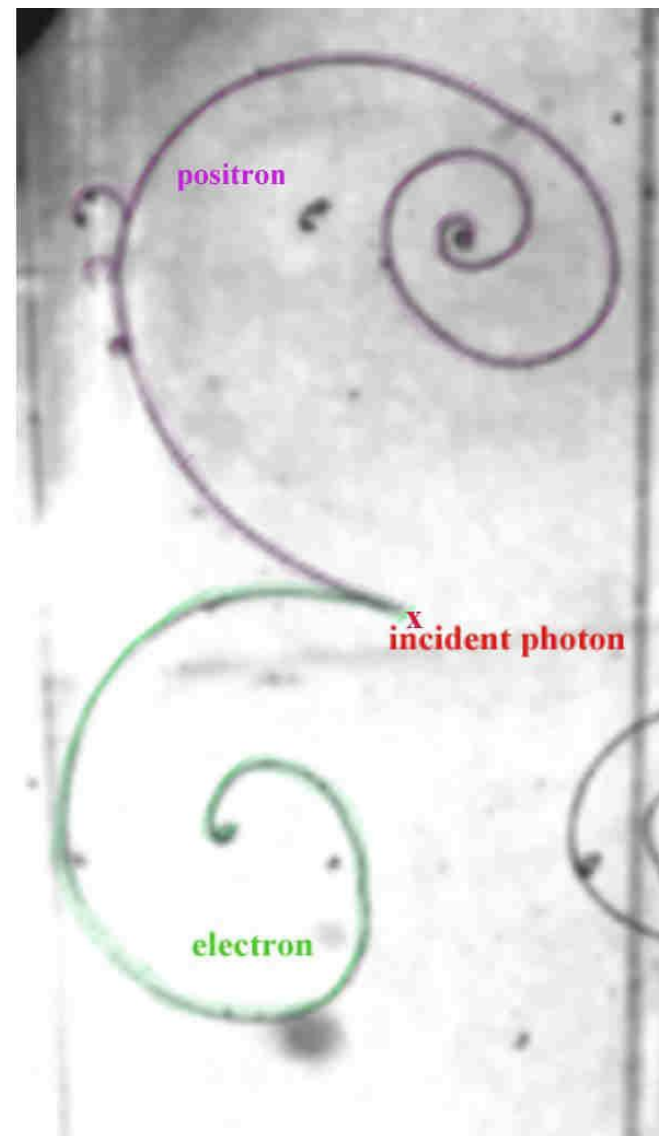
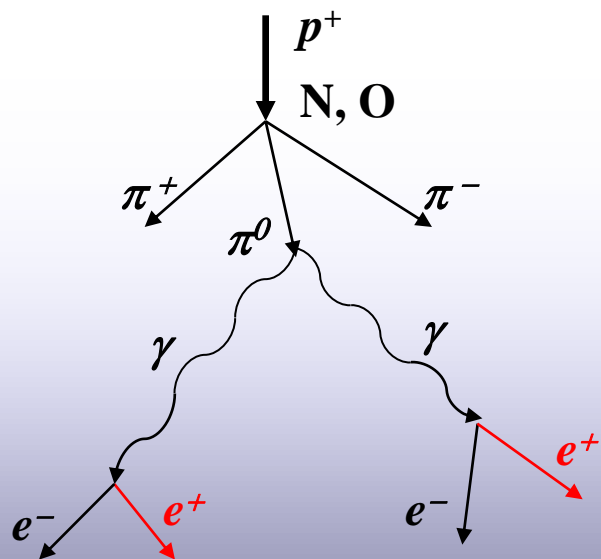
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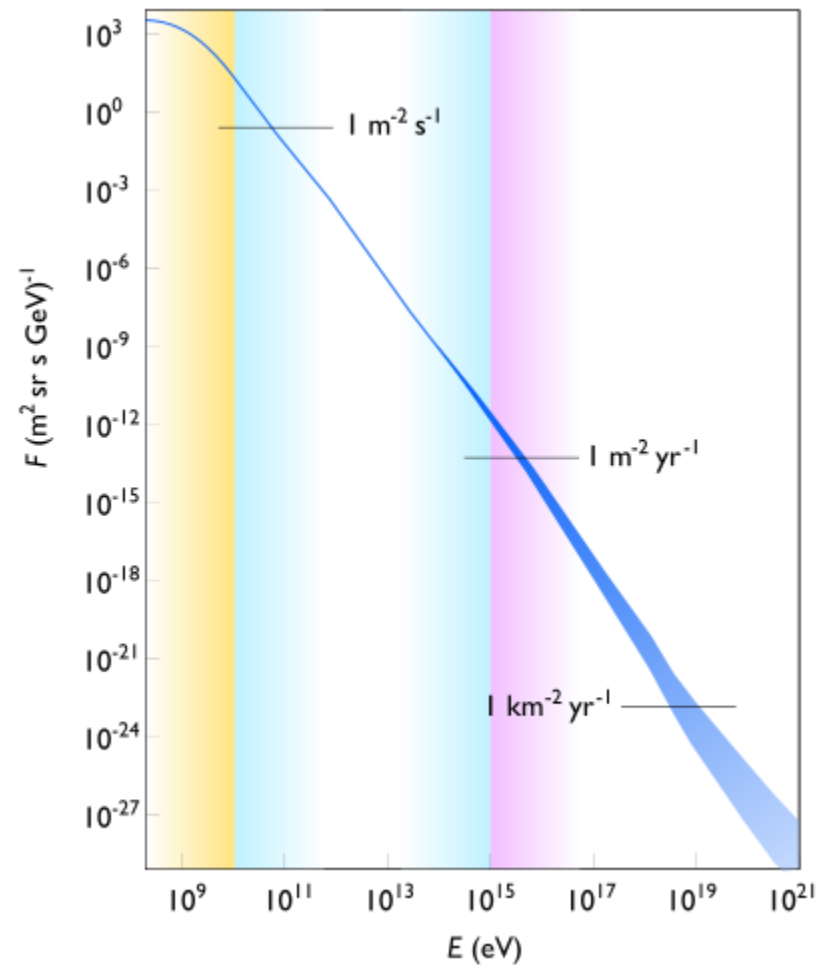
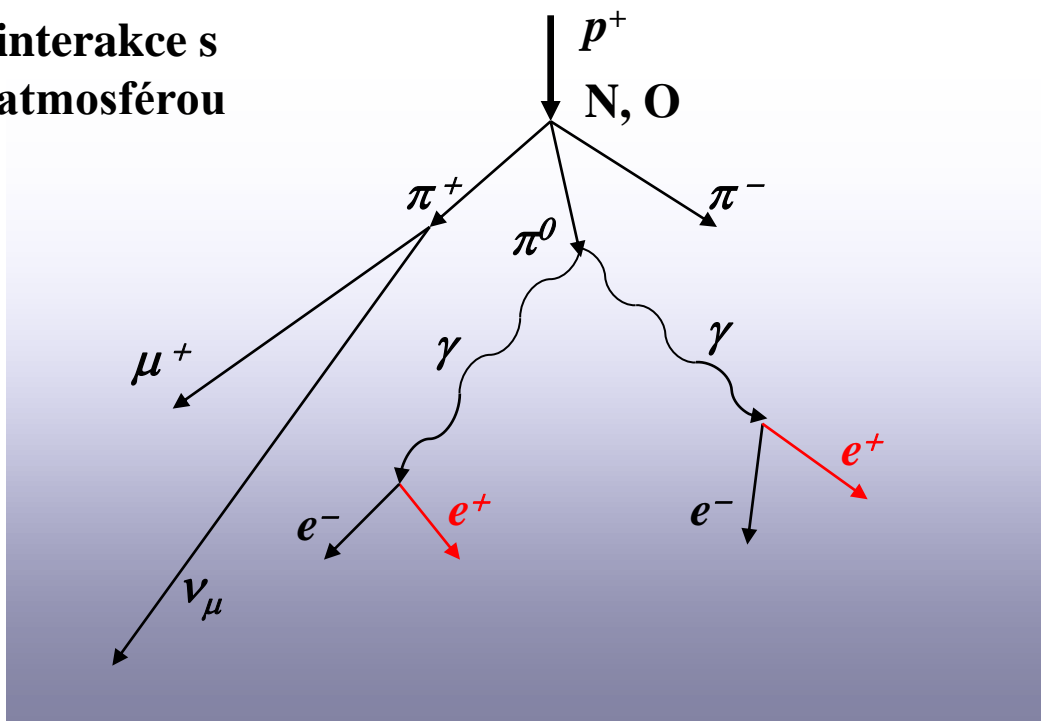
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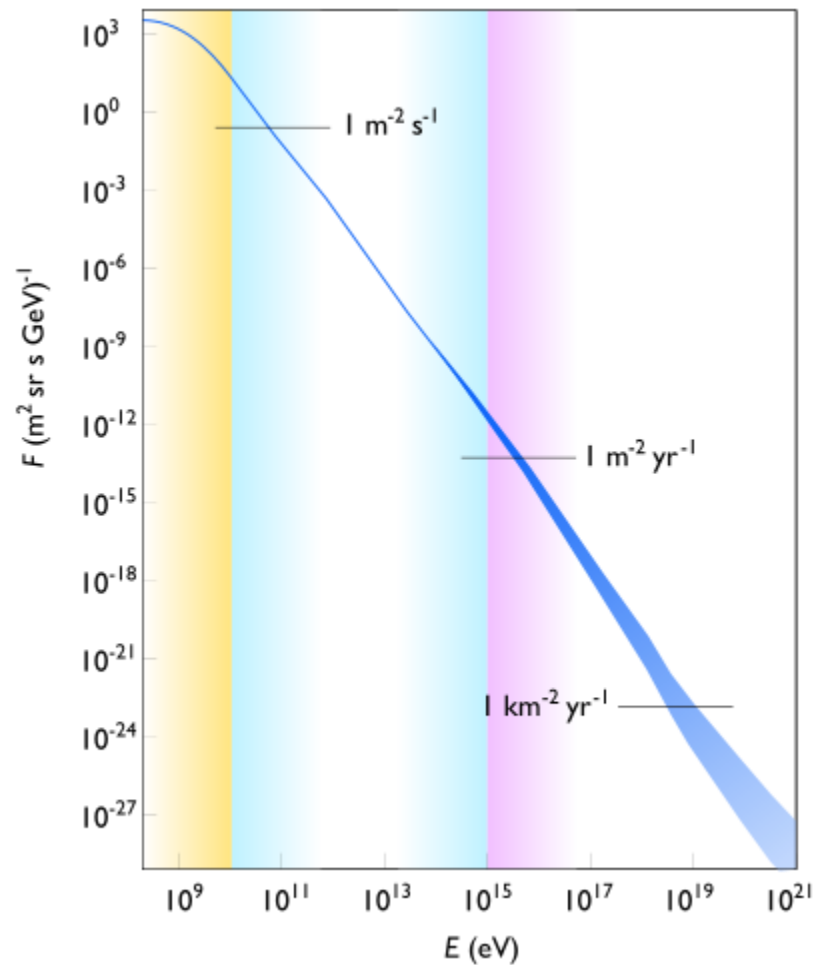
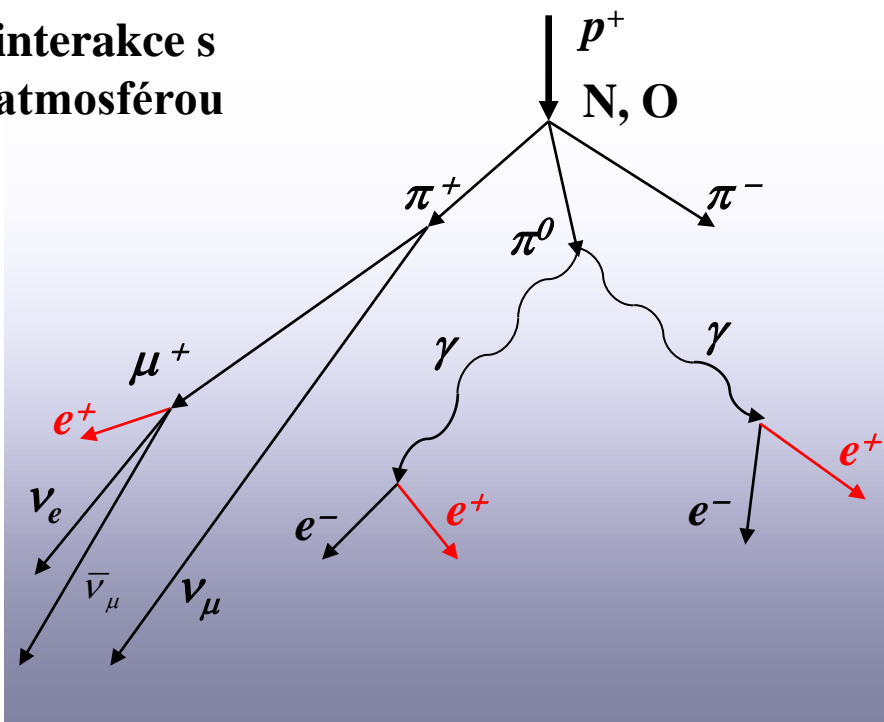
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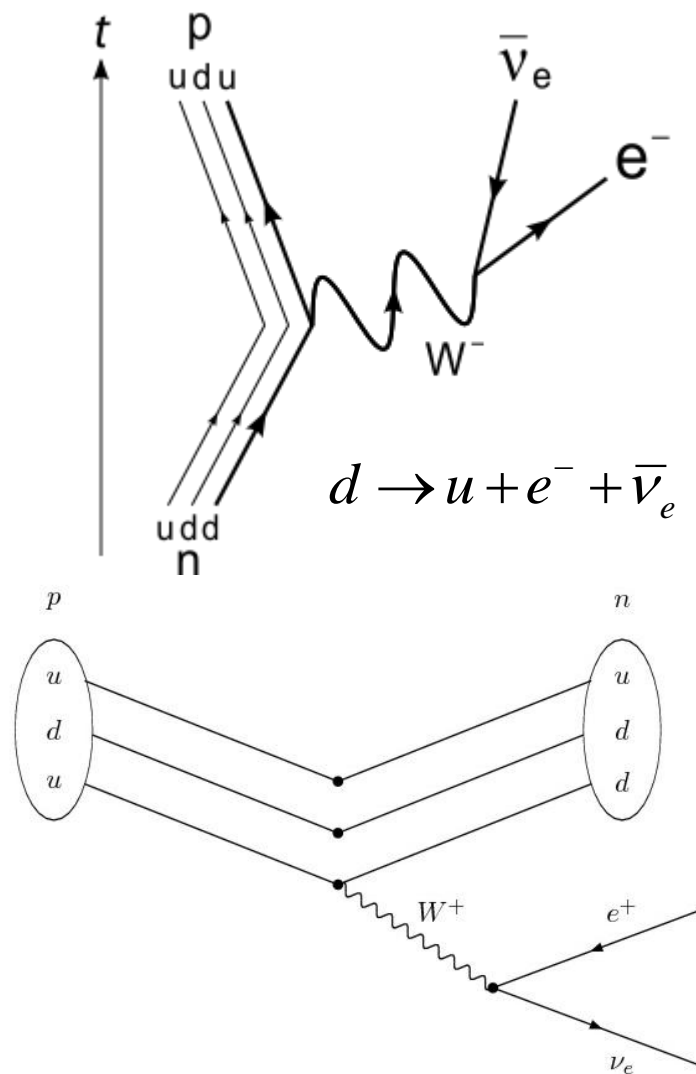
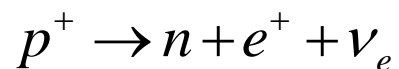
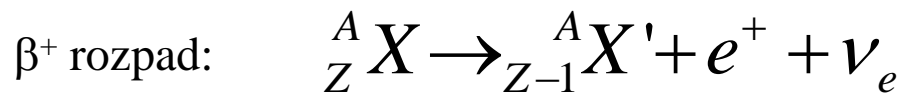
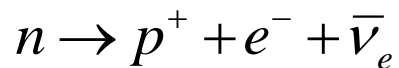
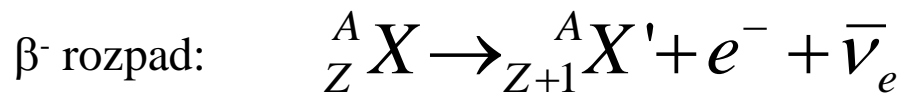
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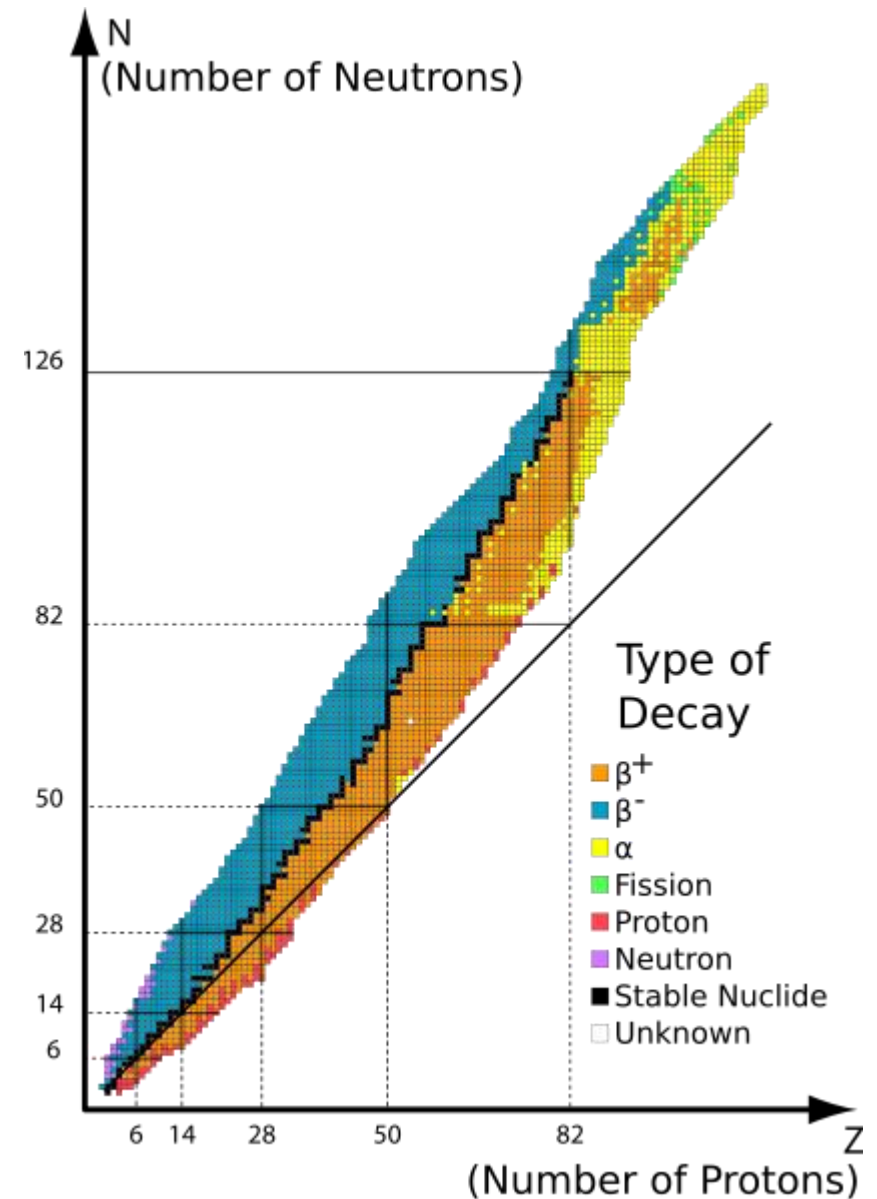
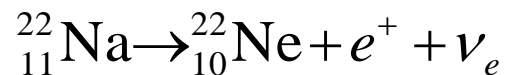
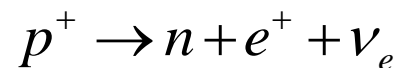
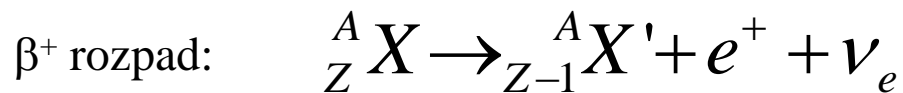
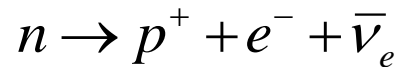
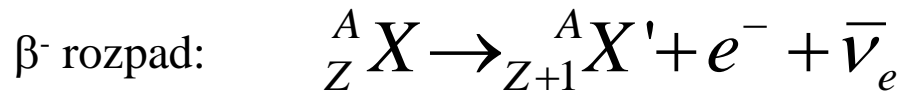
Zdroje pozitronů

β - rozpad



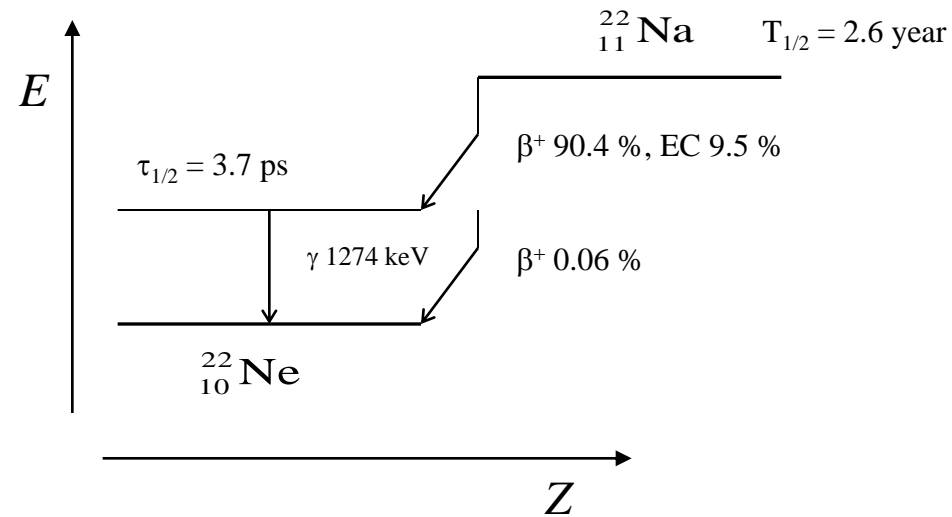
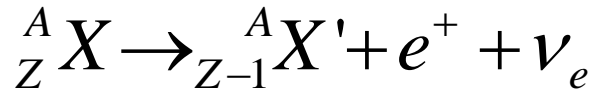
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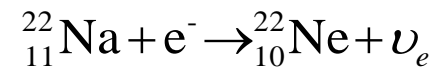
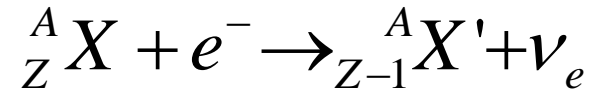


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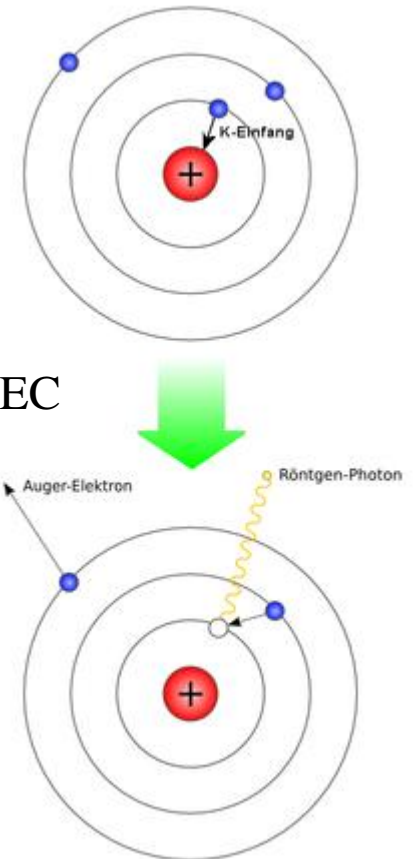
β^+ - rozpad



záchyt e^-

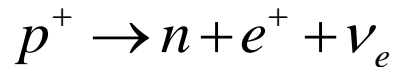
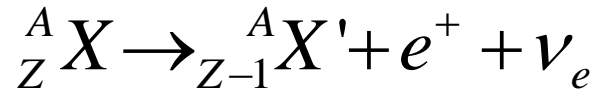


• pro $Q < 2m_e c^2$ pouze EC



Zdroje pozitronů

β^+ - rozpad

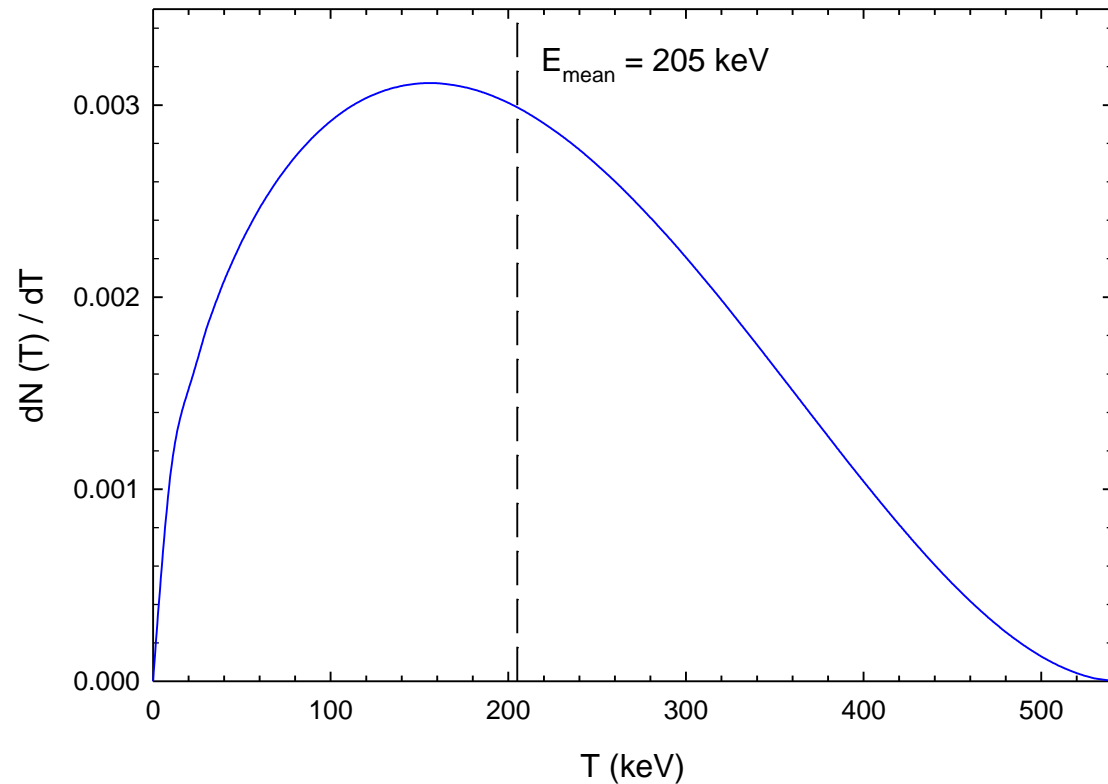


$$E_{\text{mean}} = 205 \text{ keV}$$

$$Q = E_{\text{max}} = 545 \text{ keV}$$

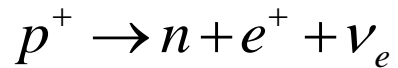
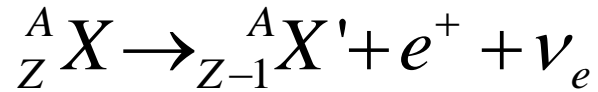
$$\frac{dN}{dT} = D(Z, Q) \sqrt{T(T + 2mc^2)} (T + mc^2) (Q - T)^2$$

energetické spektrum e^+ emitovaných ${}^{22}\text{Na}$



Zdroje pozitronů

β^+ - rozpad

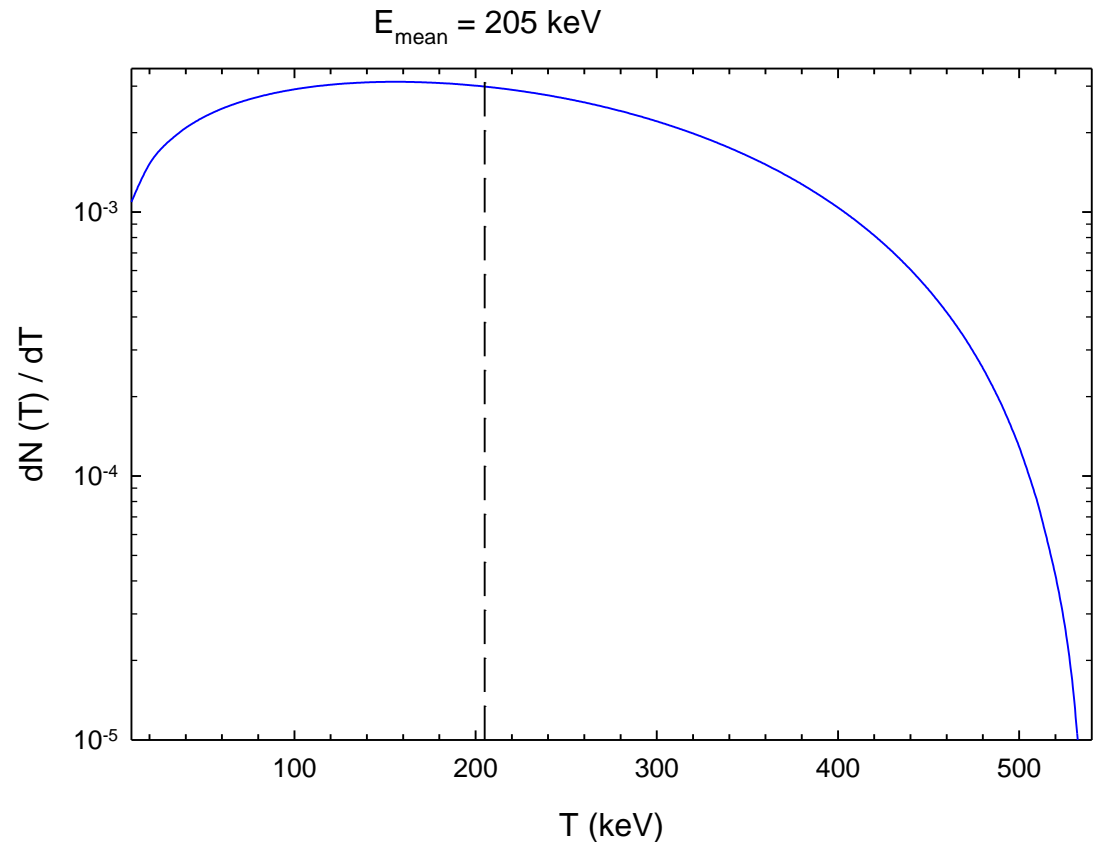


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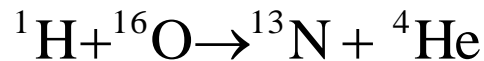
Zdroje pozitronů

β^+ - zářiče

- branching ratio (β^+ vs EC)
- E_{max} (tj. Q -value)
- poločas rozpadu $T_{1/2}$
- sekundární foton

isotope	$T_{1/2}$	e^+ yield	E_{max} (MeV)	secondary γ	E_γ (MeV)
^{13}N	9.96 min	1	1.20	0	-
^{15}O	123 s	1	1.74	0	-
^{18}F	110 min	0.97	0.64	0	-

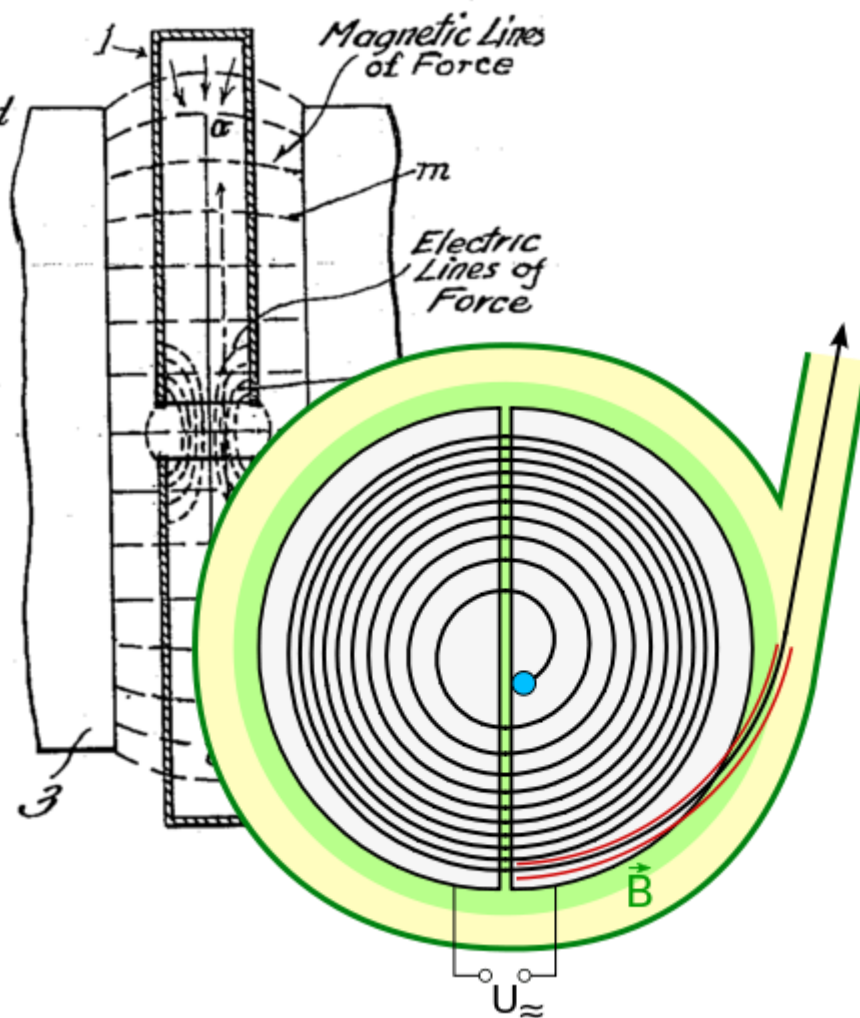
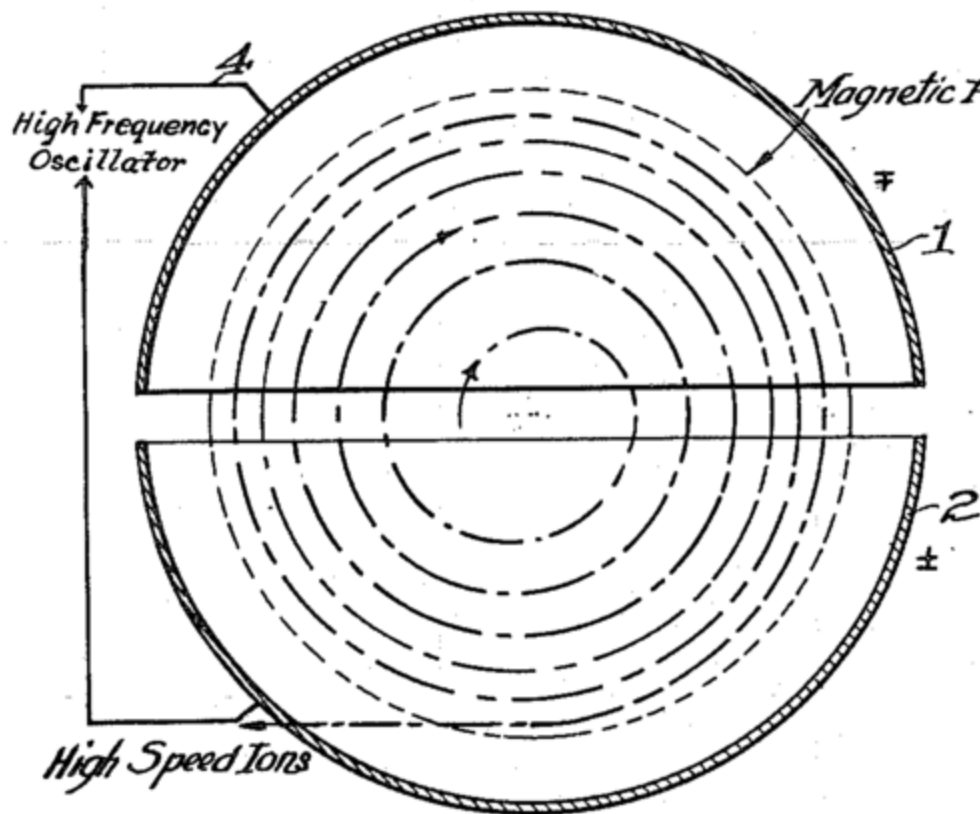
- příprava v cyklotronu



protony urychlené na $T \geq 5.2$ MeV

Zdroje pozitronů

cyklotron



Zdroje pozitronů

cyklotron



UJV Řež: cyklotron U-120M, p^+ , $T = 5.4 - 38$ MeV

Zdroje pozitronů

β^+ - zářiče

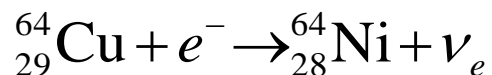
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^{13}N	9.96 min	1	1.20	0	-
^{15}O	123 s	1	1.74	0	-
^{18}F	110 min	0.97	0.64	0	-
^{22}Na	2.6 y	0.9	0.545	1	1.274
^{26}Al	8×10^5 y	0.85	1.17	1	1.81
^{44}Ti	59 y	0.98	1.47	1	1.157
^{64}Cu	12.7 h	0.178	0.653	0	-
^{68}Ge	275 d	0.88	1.90	0.02	1.078

Zdroje pozitronů

^{64}Cu

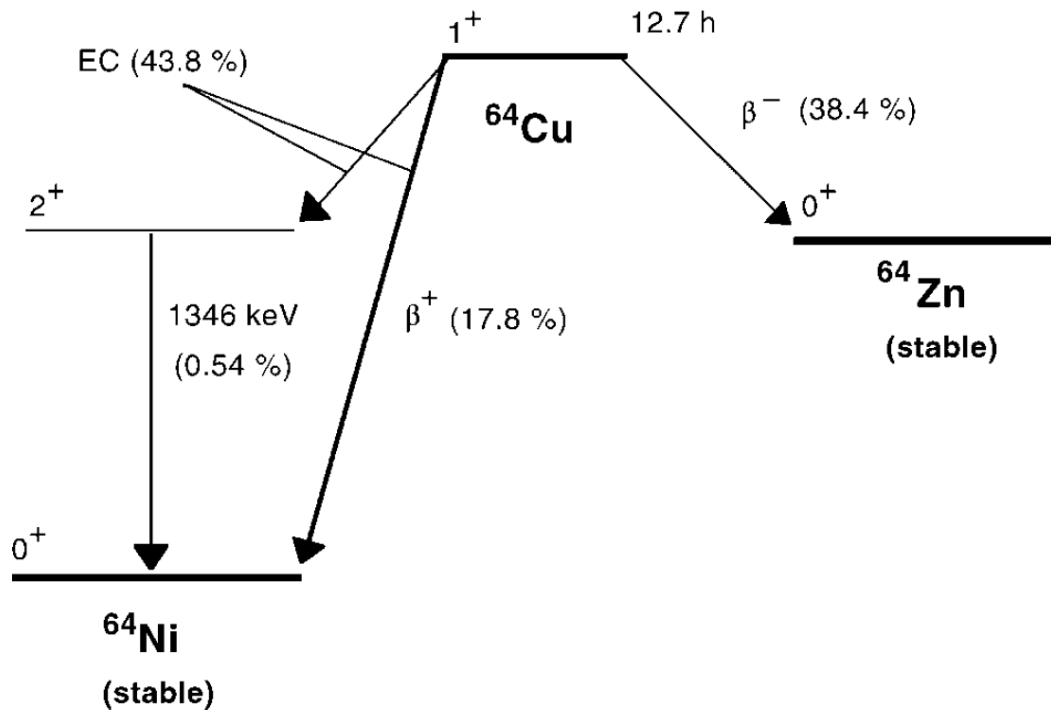
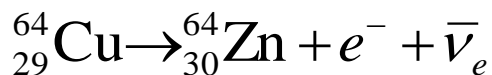
- záchyt e^- (43.8 %)



- β^+ rozpad (17.8 %)



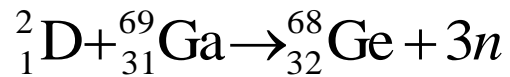
- β^- rozpad (38.4 %)



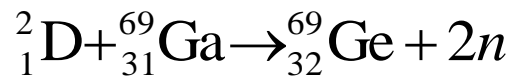
Zdroje pozitronů

příprava ^{68}Ge

cyklotron



- D ionty urychlené na $T \geq 14$ MeV
- maximální účinný průřez pro $T = 27$ MeV: $\sigma = 550$ mBarn



- pro $T = 27$ MeV $\sigma = 1650$ mBarn
- doba života ^{69}Ge je $T_{1/2} = 39$ h

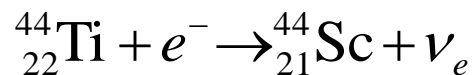


Zdroje pozitronů

$^{44}\text{Ti}/^{44}\text{Sc}$ generátor

rozpad ^{44}Ti ($T_{1/2} = 59 \text{ y}$):

- záchyt e^- (100 %)

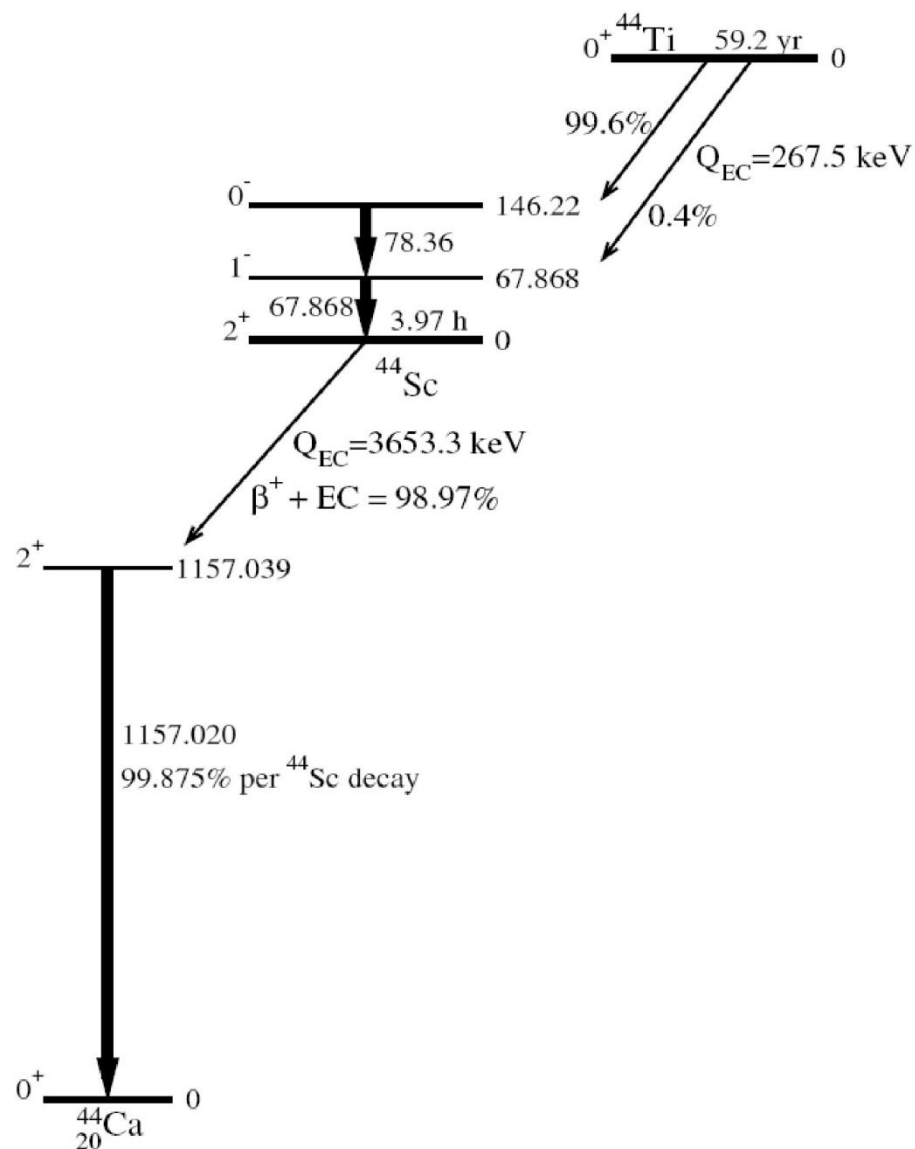


rozpad ^{44}Sc ($T_{1/2} = 3.97 \text{ h}$):

- β^+ rozpad (98 %)

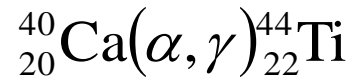


$$E_{max} = 1467 \text{ keV}$$



Zdroje pozitronů

Vznik ^{44}Ti v supernovách

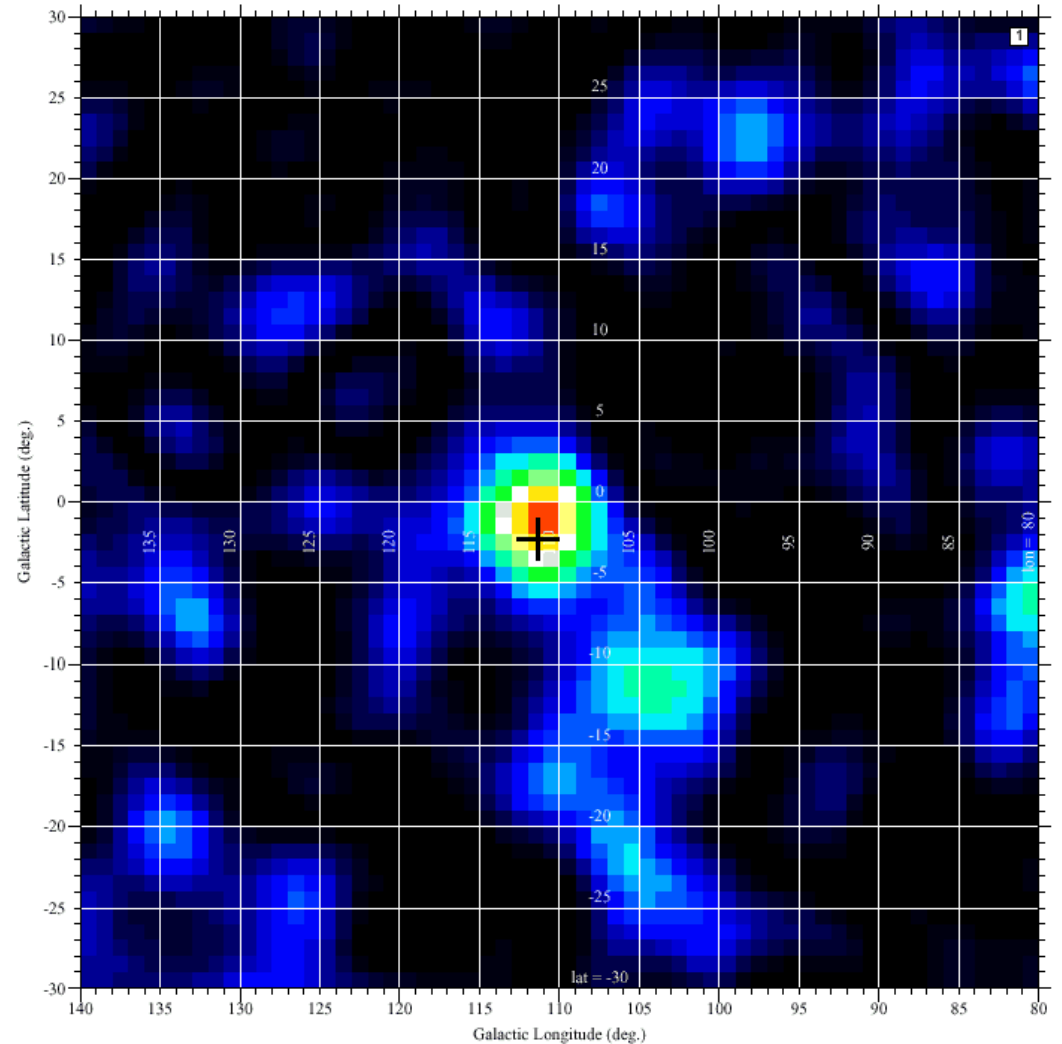


rezonance na $E_{\alpha} = 4.5 \text{ MeV}$

$$T_{1/2} = 59 \text{ y}$$



Supernova Cassiopeia A (vznik před ~ 300 lety)



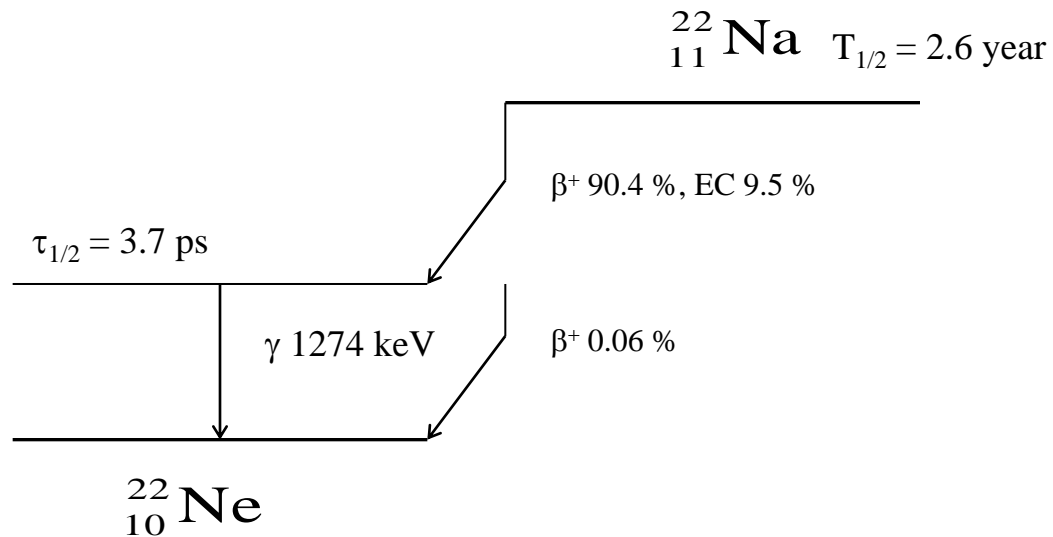
Zdroje pozitronů

^{22}Na

- β^+ rozpad, $T_{1/2} = 2.6 \text{ y}$



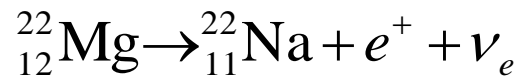
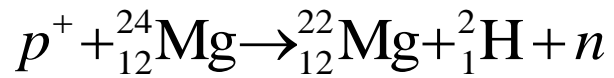
- sekundární γ 1274 MeV



Zdroje pozitronů

příprava ^{22}Na

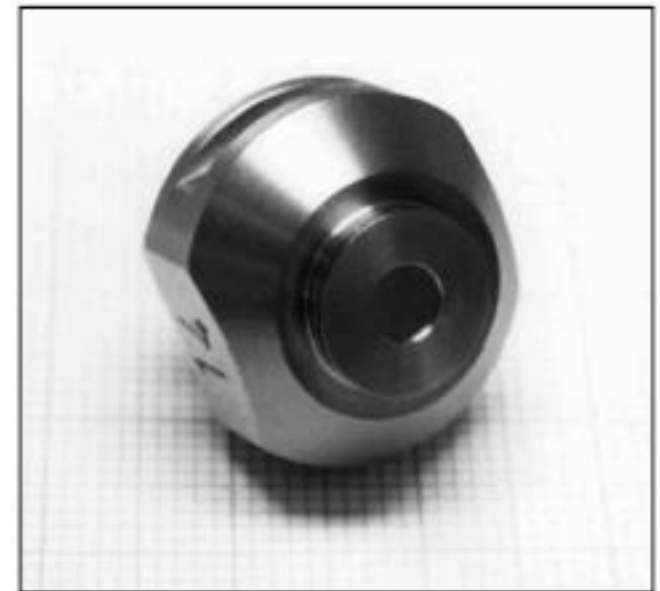
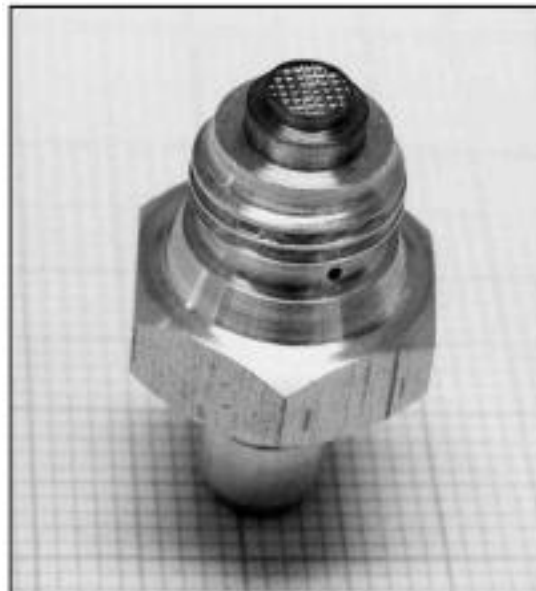
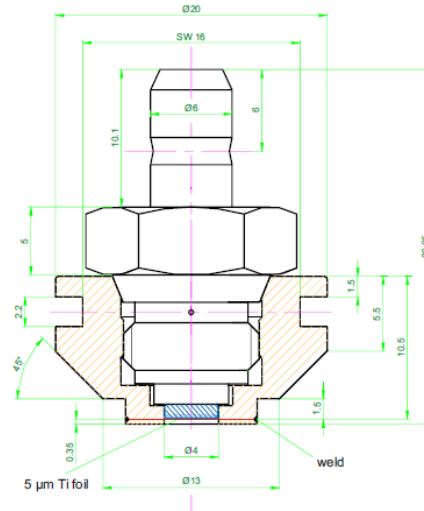
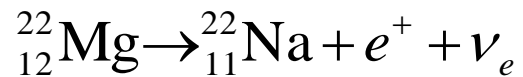
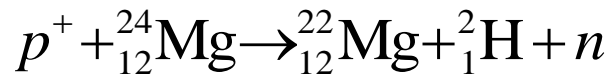
- cyklotron, p^+ , $T = 66 \text{ MeV}$



Zdroje pozitronů

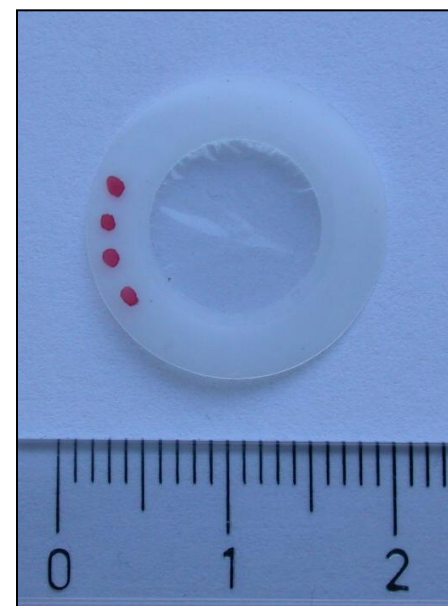
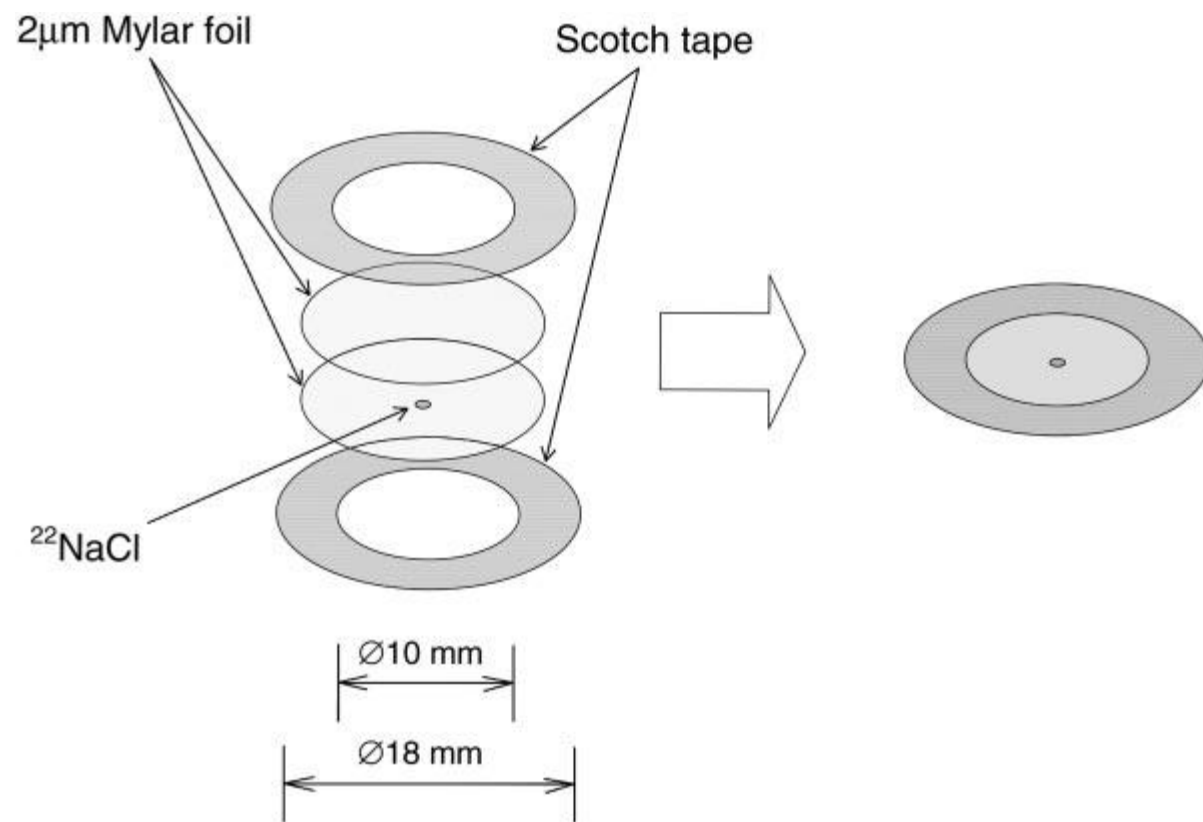
příprava ^{22}Na

- cyklotron, p^+ , $T = 66 \text{ MeV}$



Zdroje pozitronů

^{22}Na pozitronový zdroj



Hloubka průniku pozitronů

pozitrony emitované β^+ zářičem

pravděpodobnost, že pozitron pronikne do hloubky z $P(z) = \alpha e^{-\alpha z}$

$$\alpha [\text{cm}^{-1}] = 16 \frac{\rho [\text{g cm}^{-3}]}{E_{\text{max}}^{1.4} [\text{MeV}]}$$

ρ – hustota materiálu

$$E_{\text{max}} = 0.545 \text{ MeV} \quad (\text{pro } ^{22}\text{Na})$$

střední hloubka průniku $\int_0^{\infty} z P(z) dz = \frac{1}{\alpha}$

Příklad:

$$\text{Mg: } \alpha^{-1} = 154 \mu\text{m}$$

$$\text{Al: } \alpha^{-1} = 99 \mu\text{m}$$

$$\text{Cu: } \alpha^{-1} = 30 \mu\text{m}$$